Fuzzy Systems for Control Applications

Emil M. Petriu
School of Electrical Engineering and Computer Science
University of Ottawa
http://www.site.uottawa.ca/~petriu/

FUZZY SETS

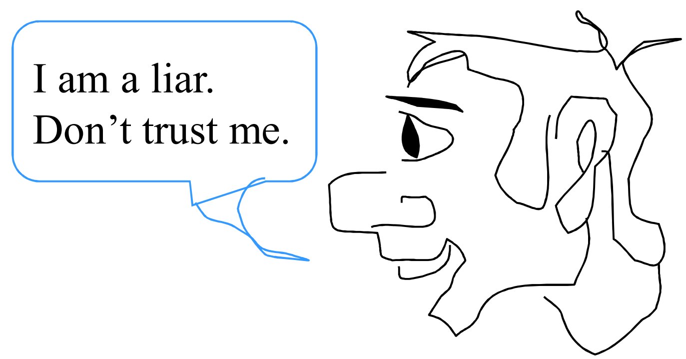
<u>Definition</u>: If **X** is a collection of objects denoted generically by **x**, then a *fuzzy set* **A** in **X** is defined as a set of ordered pairs:

$$\mathbf{A} = \{ (\mathbf{x}, \mu_A(x)) \mid x \in X \}$$

where $\mu_A(x)$ is called the *membership function* for the fuzzy set A. The membership function maps each element of X (the *universe of discourse*) to a *membership grade* between 0 and 1.

◊ Bivalent Paradox as Fuzzy Midpoint

The statement **S** and its negation $\overline{\mathbf{S}}$ have the same truth-value $\mathbf{t}\,(\mathbf{S}) = \mathbf{t}\,(\overline{\mathbf{S}})$ **.**



In the <u>binary logic</u>: $\mathbf{t}\,(\overline{\mathbf{S}}) = 1 - \mathbf{t}\,(\mathbf{S})$, and

$\mathbf{t}\,(\mathbf{S}) = 0$ or 1, ==> $\boxed{0 = 1}$!??!

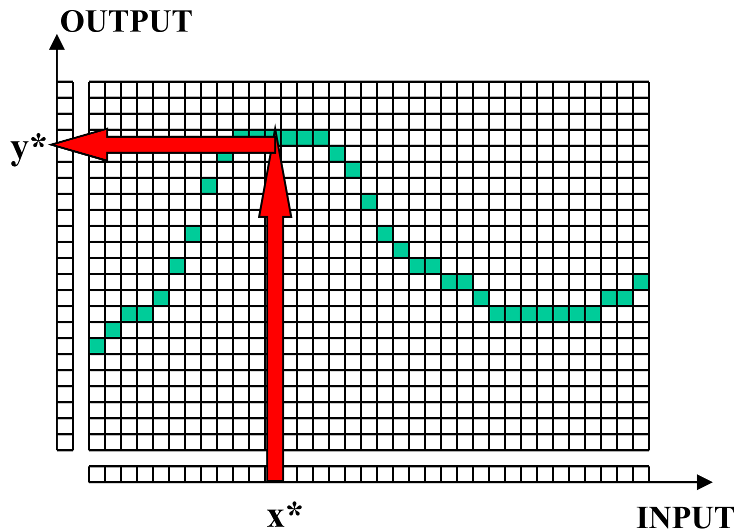
<u>Fuzzy logic</u> accepts that $\mathbf{t}\,(\mathbf{S}) = 1 - \mathbf{t}\,(\mathbf{S})$, without insisting that $\mathbf{t}\,(\mathbf{S})$ should only be 0 or 1, and accepts the **half-truth**: $\mathbf{t}\,(\mathbf{S}) = 1/2$.

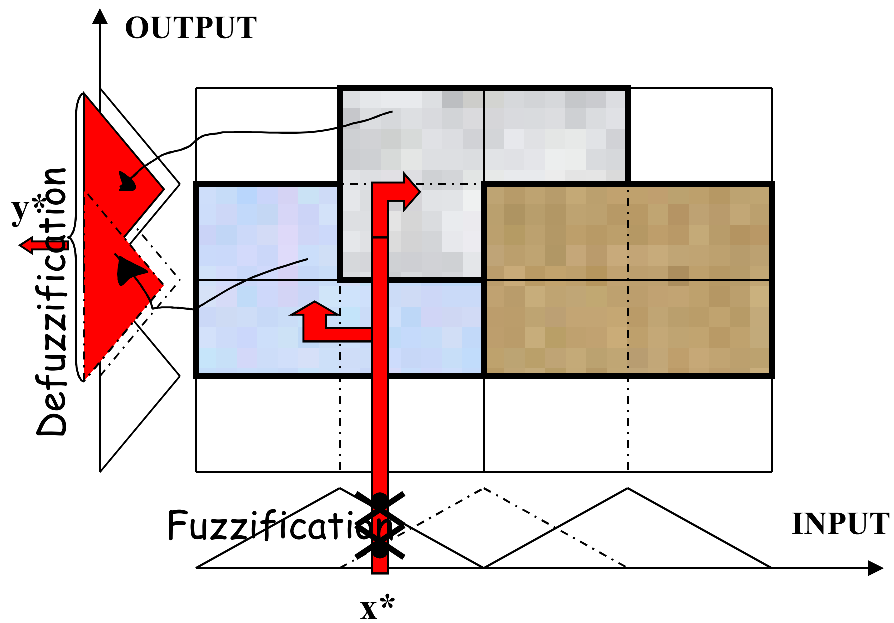
FUZZY LOGIC CONTROL

- The **basic idea of "fuzzy logic control"** (FLC) was suggested by Prof. **L.A. Zadeh**:
  - L.A. Zadeh, "A rationale for fuzzy control," *J. Dynamic Syst. Meas.Control,* vol.94, series G, pp.3-4,1972.
  - L.A. Zadeh, "Outline of a new approach to the analysis of complex systems and decision processes," *IEEE Trans. Syst., Man., Cyber.,* vol.SMC-3, no. 1, pp. 28-44, 1973.

- The **first implementation** of a FLC was reported by Mamdani and Assilian:
  - E.H. Mamdani and N.S. Assilian, "A case study on the application of fuzzy set theory to automatic control,"*Proc. IFAC Stochastic Control Symp*, Budapest, 1974.

❖ FLC provides a **nonanalytic alternative to the classical analytic control theory**. <== "But what is striking is that its most important and visible application today is in a realm not anticipated when fuzzy logic was conceived, namely, the realm of fuzzy-logic-based process control," [L.A. Zadeh, "Fuzzy logic," *IEEE Computer Mag.,* pp. 83-93, Apr. 1988].

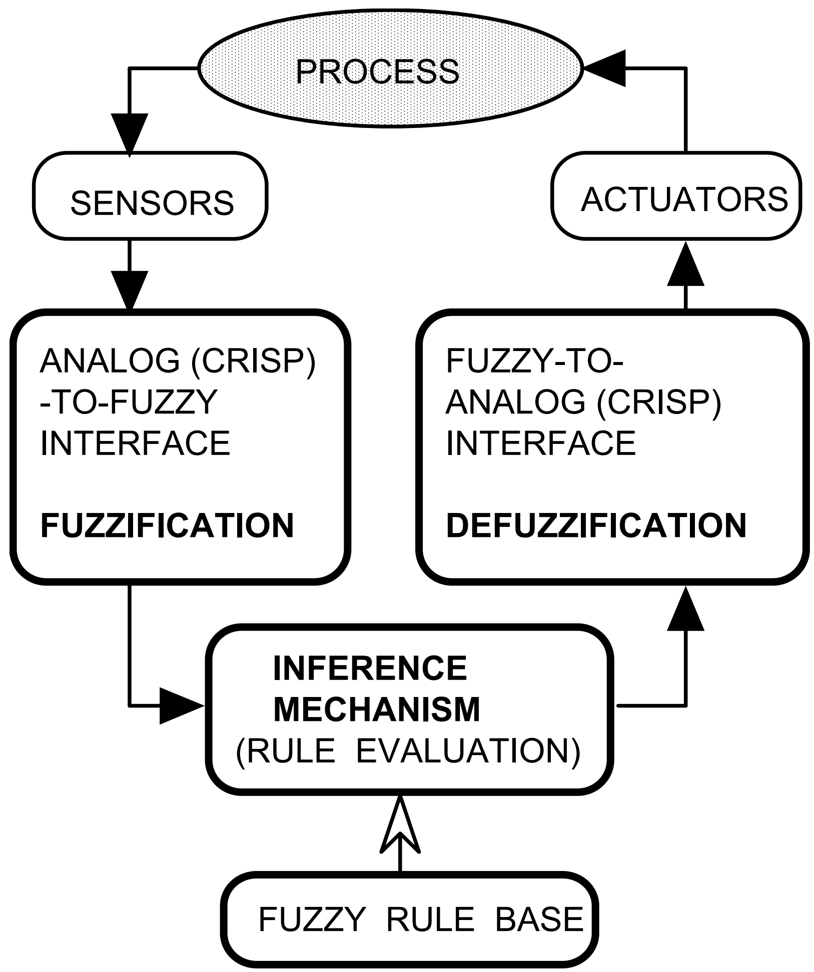




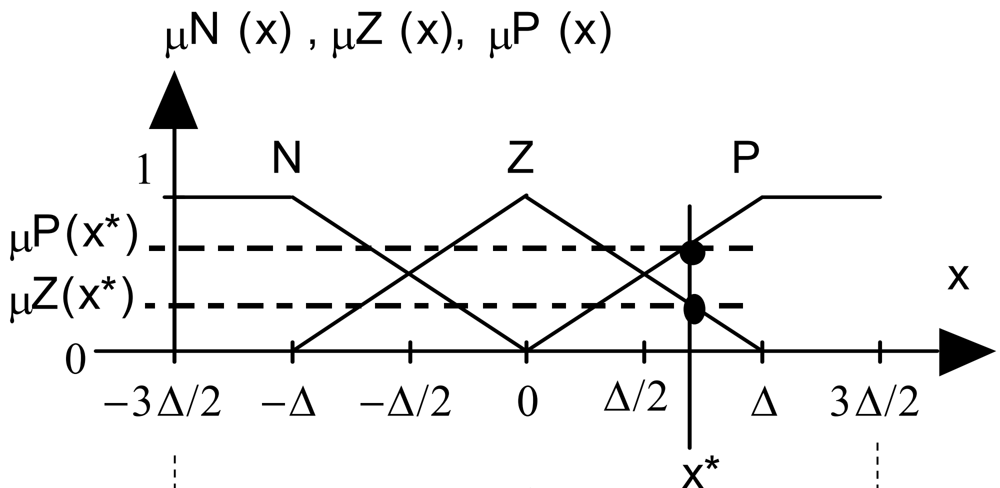
Classic control is based on a detailed I/O function OUTPUT= **F** (INPUT) which maps each high-resolution quantization interval of the input domain into a high-resolution quantization interval of the output domain.
=> Finding a mathematical expression for this detailed mapping relationship **F** may be difficult, if not impossible, in many applications.

Fuzzy control is based on an I/O function that maps each very low-resolution quantization interval of the input domain into a very low-low resolution quantization interval of the output domain. As there are only 7 or 9 fuzzy quantization intervals covering the input and output domains the mapping relationship can be very easily expressed using the"if-then" formalism. (In many applications, this leads to a simpler solution in less design time.) The overlapping of these fuzzy domains and their linear membership functions will eventually allow to achieve a rather high-resolution I/O function between crisp input and output variables.

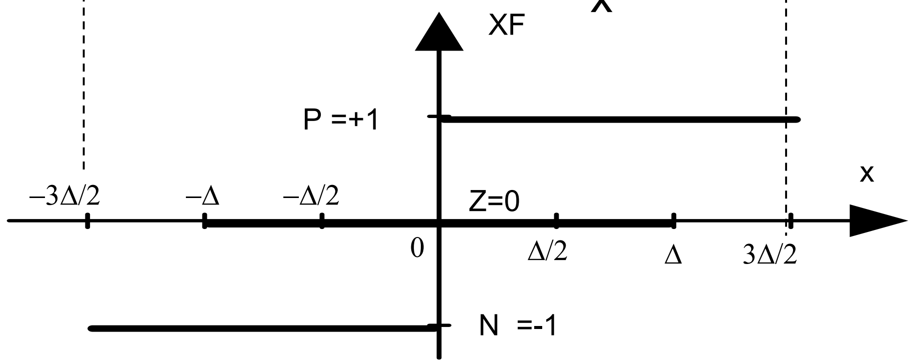
FUZZY LOGIC CONTROL



FUZZIFICATION



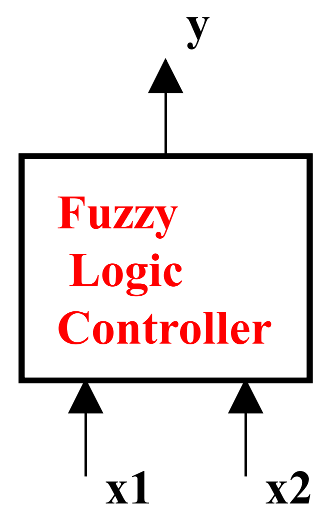
Membership functions for a 3-set fuzzy partition



Quantization characteristics for the 3-set fuzzy partition

RULE BASE:

As an example, the rule base for the two-input and one-output controller consists of a finite collection of rules with two antecedents and one consequent of the form:



$$\boldsymbol{Rule^i} : \text{if } (\mathbf{x1} \text{ is } \mathbf{A1}_{ji}) \text{ and } (\mathbf{x2} \text{ is } \mathbf{A2}_{ki}) \text{ then } (\mathbf{y} \text{ is } \mathbf{O}_m^{\,i})$$

where:

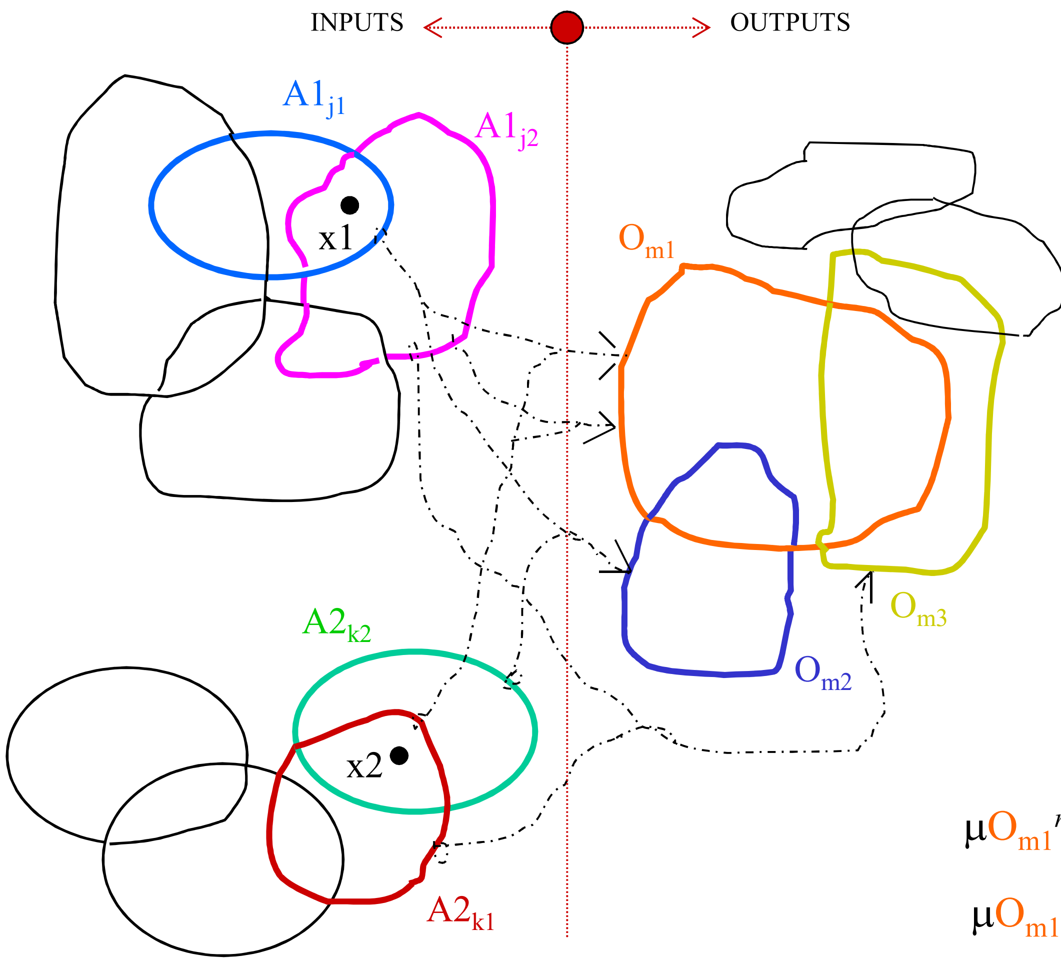
- $\mathbf{A1_j}$ is a one of the fuzzy set of the fuzzy partition for **x1**
- $\mathbf{A2_k}$ is a one of the fuzzy set of the fuzzy partition for **x2**
- $\mathbf{O_m^{\,i}}$ is a one of the fuzzy set of the fuzzy partition for **y**

- For a given pair of crisp input values *x1* and *x2* the antecedents are the degrees of membership obtained during the fuzzification: $\mu A1_j(x1)$ and $\mu A2_k(x2)$. The strength of the ***Rule***^{i} (i.e its impact on the outcome) is as strong as its weakest component:

$$\mu O_m^{\,i}(y) = \mathbf{min}\ [\mu A1_{ji}(x1),\ \mu A2_{ki}(x2)]$$

- If more than one activated rule, for instance ***Rule*** ^{p} and ***Rule*** ^{q}, specify the same output action, (e.g. y is O_m), then the strongest rule will prevail:

$$\mu O_m^{\,p\&q}(y) = \mathbf{max}\ \{\ \mathbf{min}[\mu A1_{jp}(x1),\ \mu A2_{kp}(x2)],\ \mathbf{min}[\mu A1_{jq}(x1),\ \mu A2_{kq}(x2)]\ \}$$



x1	x2	RULE	y
$A1_{j1}$	$A2_{k1}$	*r1*	O_{m1}
$A1_{j1}$	$A2_{k2}$	*r2*	O_{m2}
$A1_{j2}$	$A2_{k1}$	*r3*	O_{m1}
$A1_{j2}$	$A2_{k2}$	*r4*	O_{m3}

$$\mu O_{m1}{}^{r1}(y)=\mathbf{min}\ [\mu A1_{j1}(x1),\ \mu A2_{k1}(x2)]$$

$$\mu O_{m1}{}^{r3}(y)=\mathbf{min}\ [\mu A1_{j2}(x1),\ \mu A2_{k1}(x2)]$$

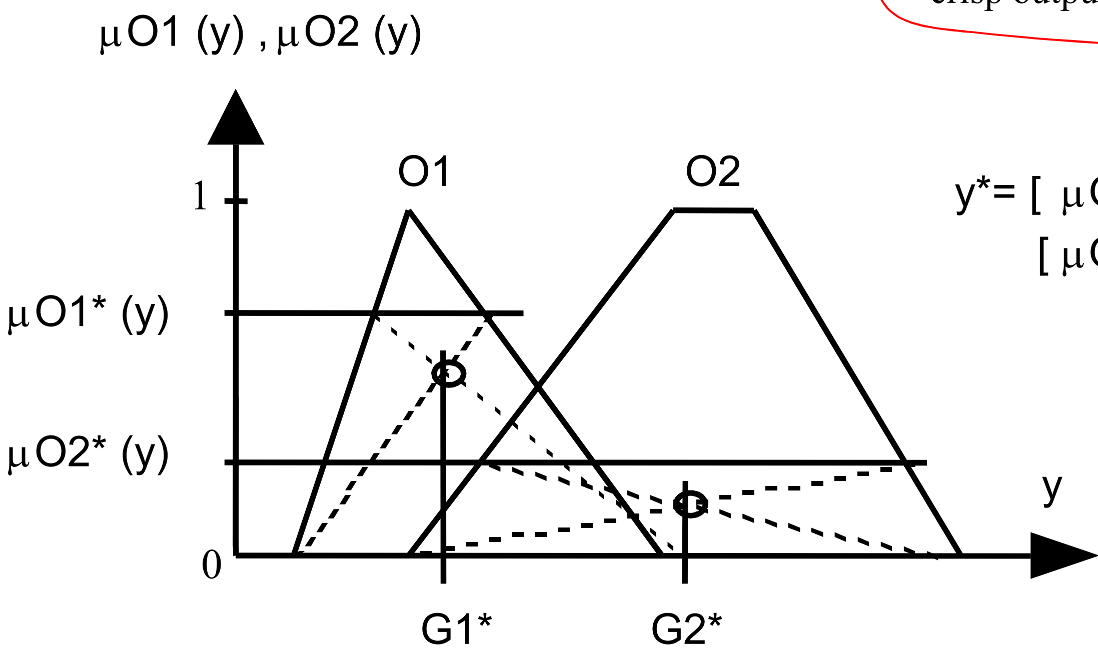
$$\boxed{\mu O_{m1}{}^{r1\,\&\,r3}(y)=\mathbf{max}\ \{\mathbf{min}[\mu A1_{j1}(x1),\ \mu A2_{k1}(x2)],\ \mathbf{min}[\mu A1_{j2}(x1),\ \mu A2_{k1}(x2)]\}}$$

$$\boxed{\mu O_{m2}{}^{r2}(y)=\mathbf{min}\ [\mu A1_{j1}(x1),\ \mu A2_{k2}(x2)]}$$

$$\boxed{\mu O_{m3}{}^{r4}(y)=\mathbf{min}\ [\mu A1_{j2}(x1),\ \mu A2_{k2}(x2)]}$$

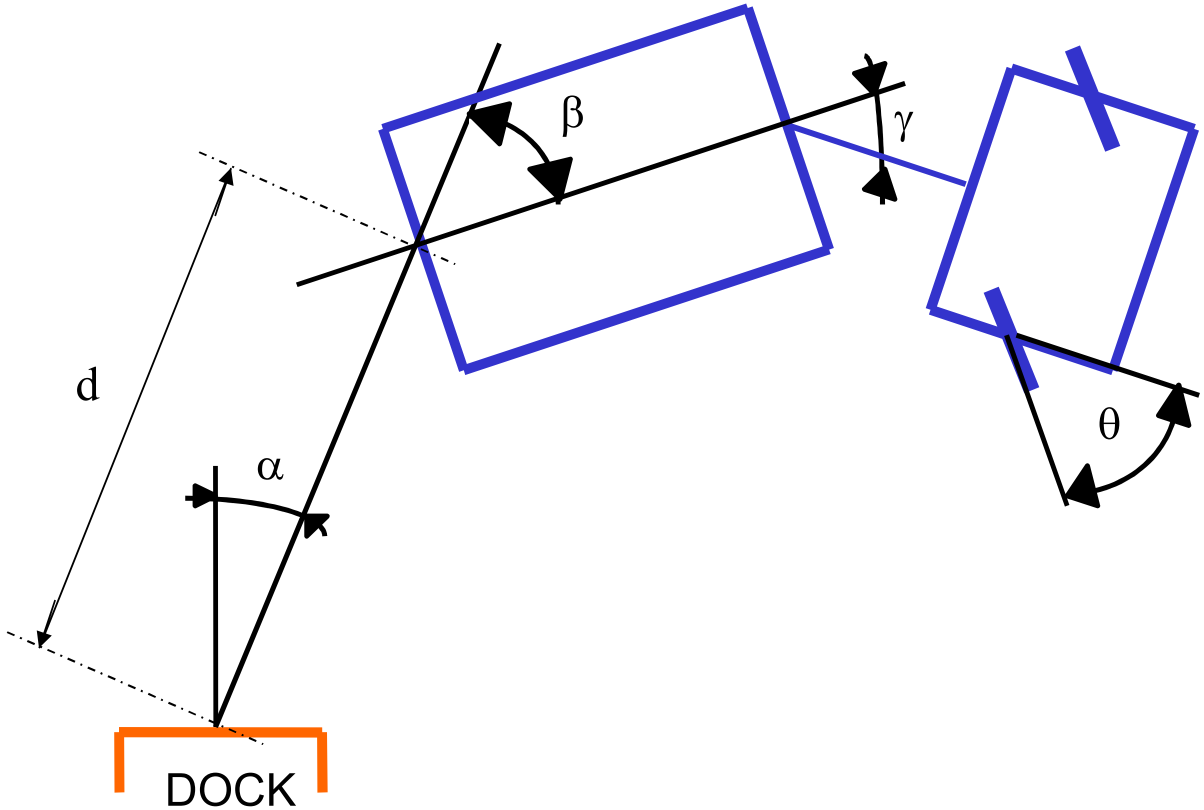
DEFUZZIFICATION

Center of gravity (COG) defuzzification method avoids the defuzzification ambiguities which may arise when an output degree of membership can come from more than one crisp output value



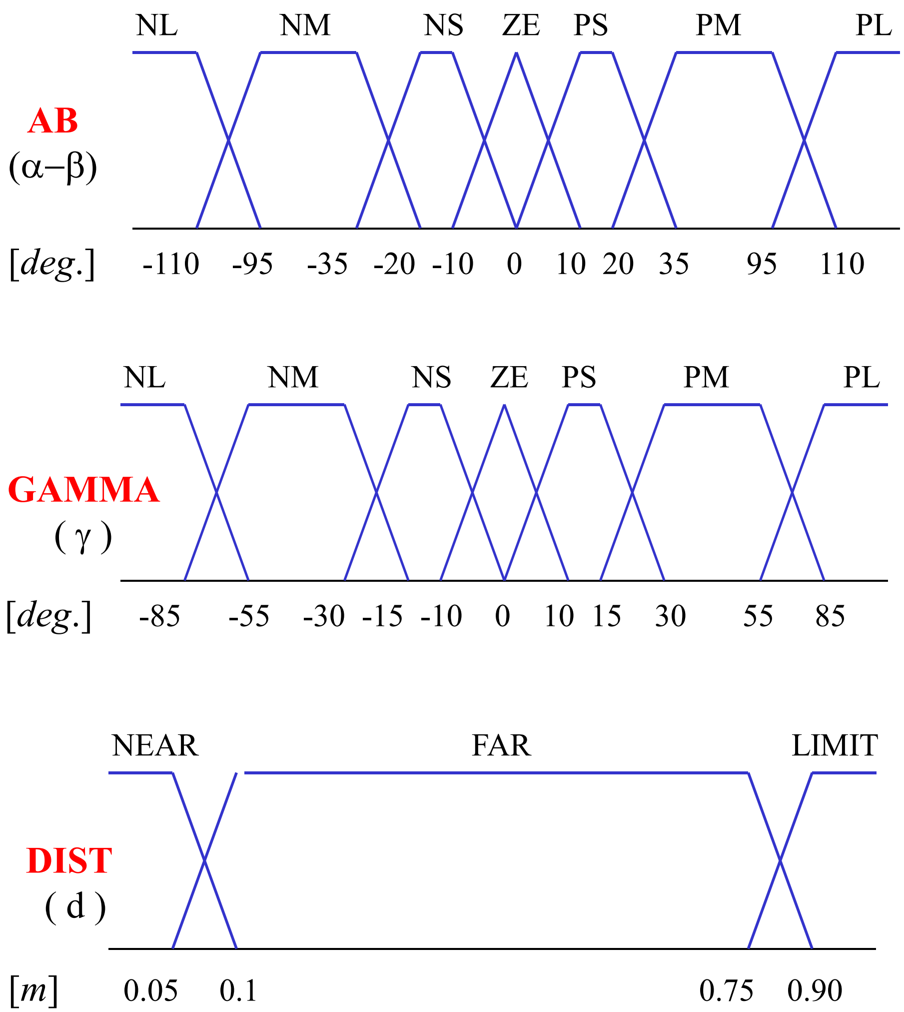
$$y^* = [\mu O1^*(y) \cdot G1^* + \mu O2^*(y) \cdot G2^*] / [\mu O1^*(y) + \mu O2^*(y)]$$

Fuzzy Controller for Truck and Trailer Docking

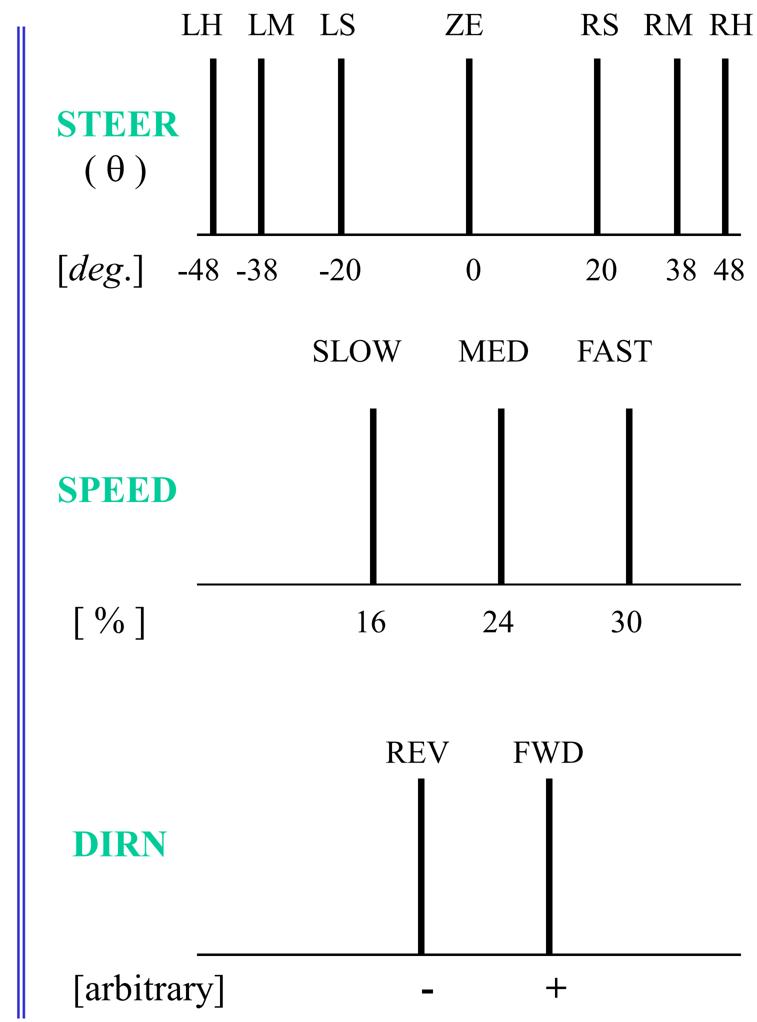


>>> Truck & trailer docking

INPUT MEMBERSHIP FUNCTIONS



OUTPUT MEMBERSHIP FUNCTIONS



STEER / DIRN

rule base

AB ($\alpha-\beta$) — **GAMMA** (γ)

AB \ GAMMA	*NL*	*NM*	*NS*	*ZE*	*PS*	*PM*	*PL*
NL	LH/F	LH/F	LH/F	LM/F	LS/F	RS/F	RM/F
NM	LH/F	LH F-R	LH F-R	LM F-R	ZE F-R	RM F-R	RH/F
NS	LH/F	LH F-R	LM/R	LS/R	RS/R	RM F-R	RH/F
ZE	LH/F	LH F-R	LS/R	ZE/R	RS/R	RH F-R	RH/F
PS	LH/F	LM F-R	LS/R	RS/R	RM/R	RH F-R	RH/F
PM	LH/F	LM F-R	ZE F-R	RM F-R	RH F-R	RH F-R	RH/F
PL	LM/F	LS/F	RS/F	RM/F	RH/F	RH/F	RH/F

There is a ***hysteresis ring*** around the center of the rule base table for the **DIRN** output. This means that when the vehicle reaches a state within this ring, it will continue to drive in the same direction, **F** (forward) or **R** (reverse), as it did in the previous state outside this ring.

The hysteresis was purposefully introduced to increase the robustness of the FLC.

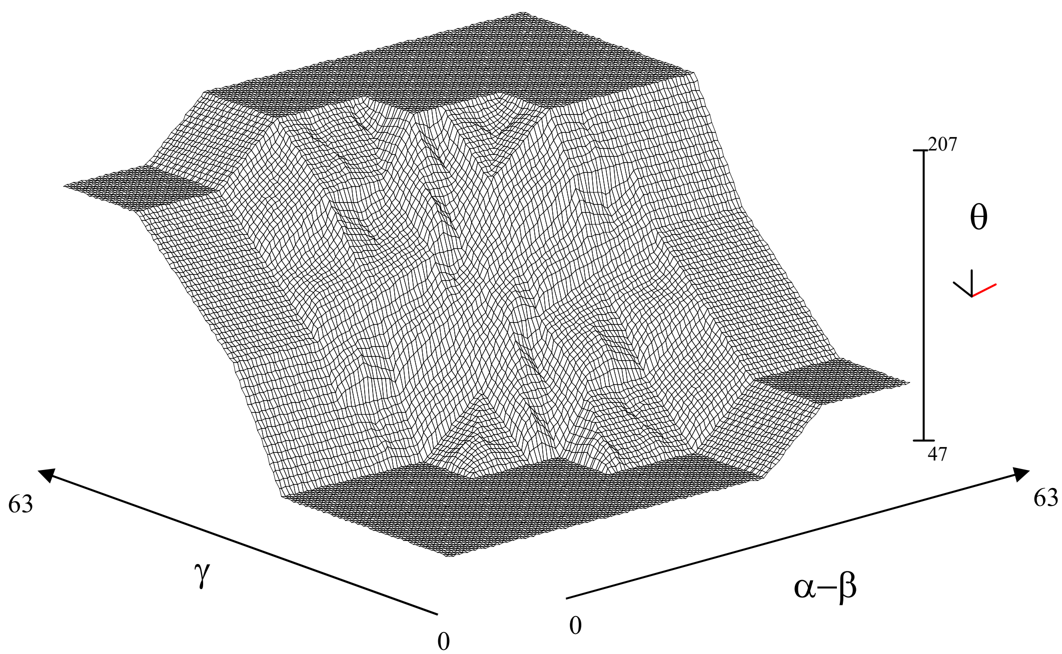
DEFFUZIFICATION

- The crisp value of the steering angle is obtained by the modified "centroidal" deffuzification (Mamdani inference):

$$\theta = (\mu_{LH} \cdot \theta_{LH} + \mu_{LM} \cdot \theta_{LM} + \mu_{LS} \cdot \theta_{LS} + \mu_{ZE} \cdot \theta_{ZE} + \mu_{RS} \cdot \theta_{RS} + \mu_{RM} \cdot \theta_{RM} + \mu_{RH} \cdot \theta_{RH}) / (\mu_{LH} + \mu_{LM} + \mu_{LS} + \mu_{ZE} + \mu_{RS} + \mu_{RM} + \mu_{RL})$$

μ_{XX} is the current membership value (obtained by a "*max-min*" compositional mode of inference) of the output θ to the fuzzy class XX, where

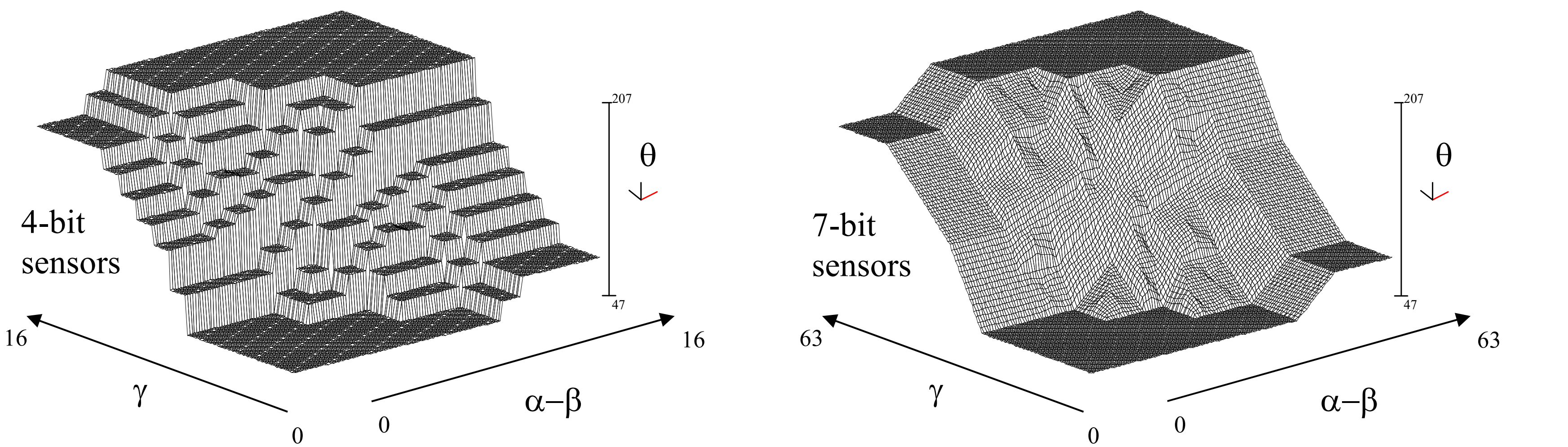
$XX \in$ {LH, LM, LS, ZE, RS, RM, RH}.



I/O characteristic of the Fuzzy Logic Controller for truck and trailer docking.

"FUZZY UNCERTAINTY" ==> WHAT ACTUALLY IS "FUZZY" IN A FUZZY CONTROLLER ??

- There is tenet of common wisdom that FLCs are meant to successfully deal with uncertain data. According to this, FLCs are supposed to make do with "uncertain" data coming from (cheap) low-resolution and imprecise sensors. However, experiments show that the **low resolution of the sensor data results in rough quantization of of the controller's I/O characteristic**:



I/O characteristics of the FLC for truck & trailer docking for 4-bit sensor data (α, β, γ) and 7-bit sensor data.

- The key benefit of FLC is that the desired system behavior can be described with simple "if-then" relations based on very low-resolution models able to incorporate **empirical** (i.e. not too **"certain"?**) **engineering knowledge**. FLCs have found many practical applications in the context of **complex ill-defined processes that can be controlled by skilled human operators**: water quality control, automatic train operation control, elevator control, nuclear reactor control, automobile transmission control, etc.,

Fuzzy Control for Backing-up a Four Wheel Truck

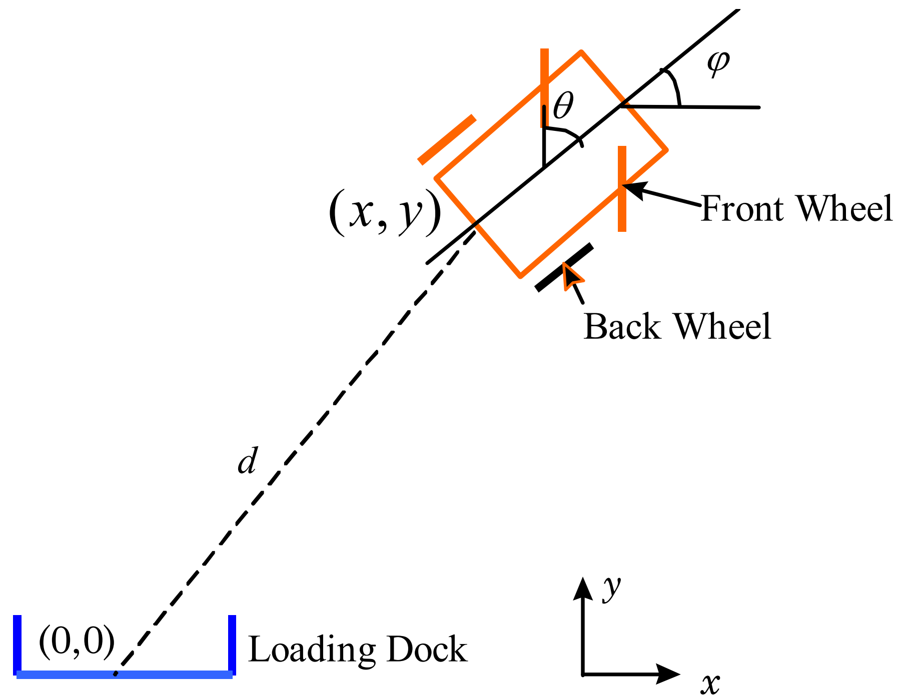
- Using a truck backing-up Fuzzy Logic Controller (FLC) as test bed, this Experiment revisits a **tenet of common wisdom** which considers FLCs as beingmeant to **make do with** uncertain data coming from **low-resolution sensors**.

- The experiment studies the effects of the input sensor-data resolution on the I/O characteristics of the digital FLC for backing-up a four-wheel truck.

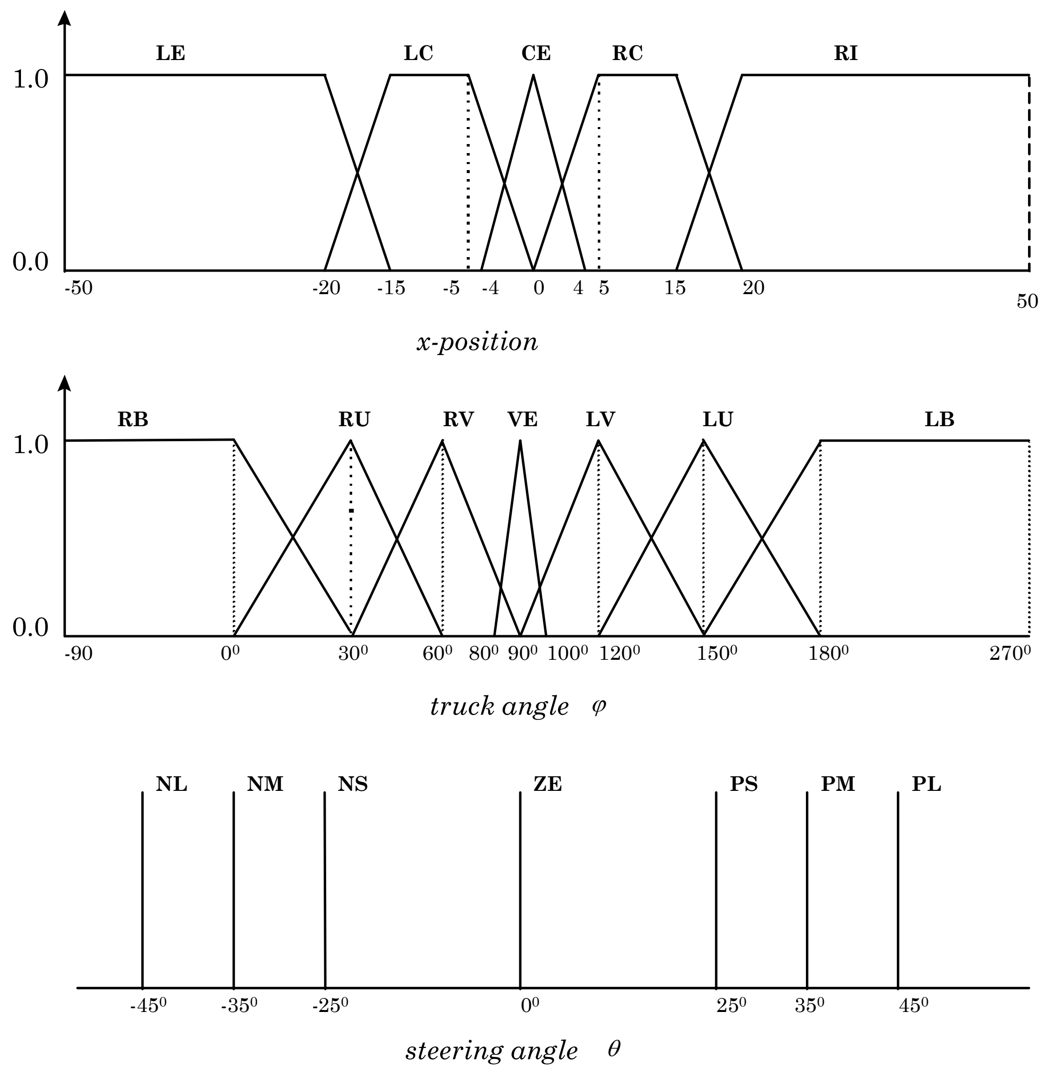
- Simulation experiments have shown that the low resolution of the sensor data results in a rough quantization of the controller's I/O characteristic. They also show that it is possible to **smooth the I/O characteristic** of a digital FLC by **dithering** the sensor data before quantization.

The truck backing-up problem

Design a Fuzzy Logic Controller (FLC) able to back up a truck into a docking station from any initial position that has enough clearance from the docking station.



Membership functions for the truck backer-upper FLC



The FLC is based on the Sugeno-style fuzzy inference.

The fuzzy rule base consists of 35 rules.

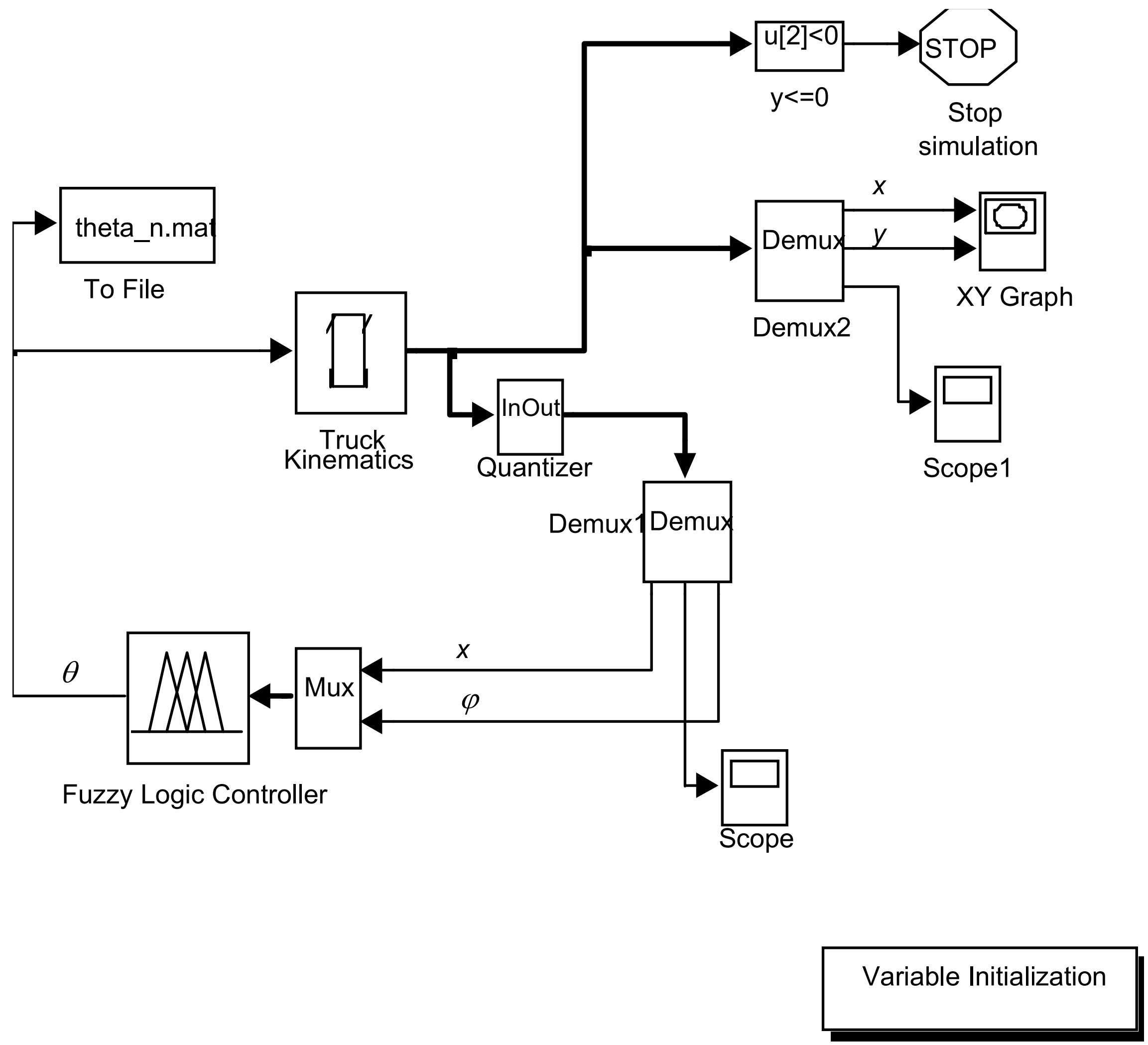
φ \ x	LE	LC	CE	RC	RI
RL	NL [1]	NL [2]	NM [3]	NM [4]	NS [5]
RU	NL [6]	NL [7]	NM	NS	PS
RV	NL	NM	NS	PS	PM
VE	NM	NM	ZE [18]	PM	PM
LV	NM	NS	PS	PM	PL
LU	NS	PS	PM	PL	PL [30]
LL	PS [31]	PM [32]	PM [33]	PL [34]	PL [35]

- *Matlab*-*Simulink* to model different FLC scenarios for the truck backing-up problem. The initial state of the truck can be chosen anywhere within the 100-by-50 experiment area as long as there is enough clearance from the dock. The simulation is updated every 0.1 s. The truck stops when it hits the loading dock situated in the middle of the bottom wall of the experiment area.

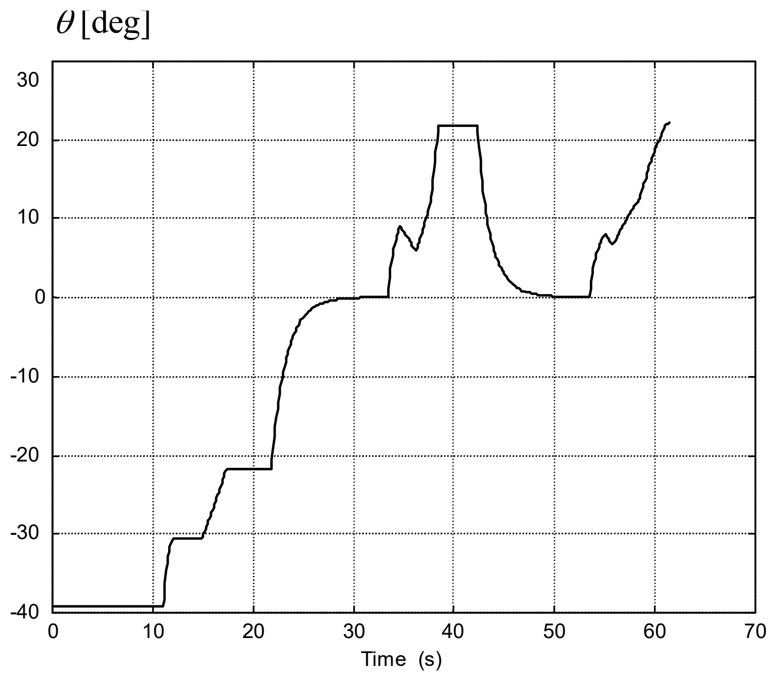
- The *Truck Kinematics* model is based on the following system of equations:

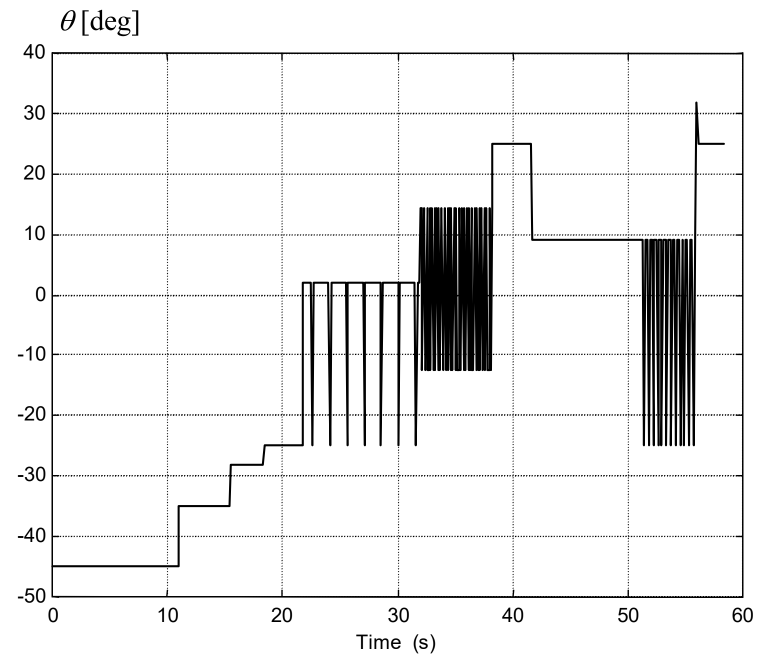
$$\begin{cases} x = -v\cos(\varphi) \\ y = -v\sin(\varphi) \\ \varphi = -\dfrac{v}{l}\sin(\theta) \end{cases}$$

where v is the backing up speed of the truck and l is the length of the truck.

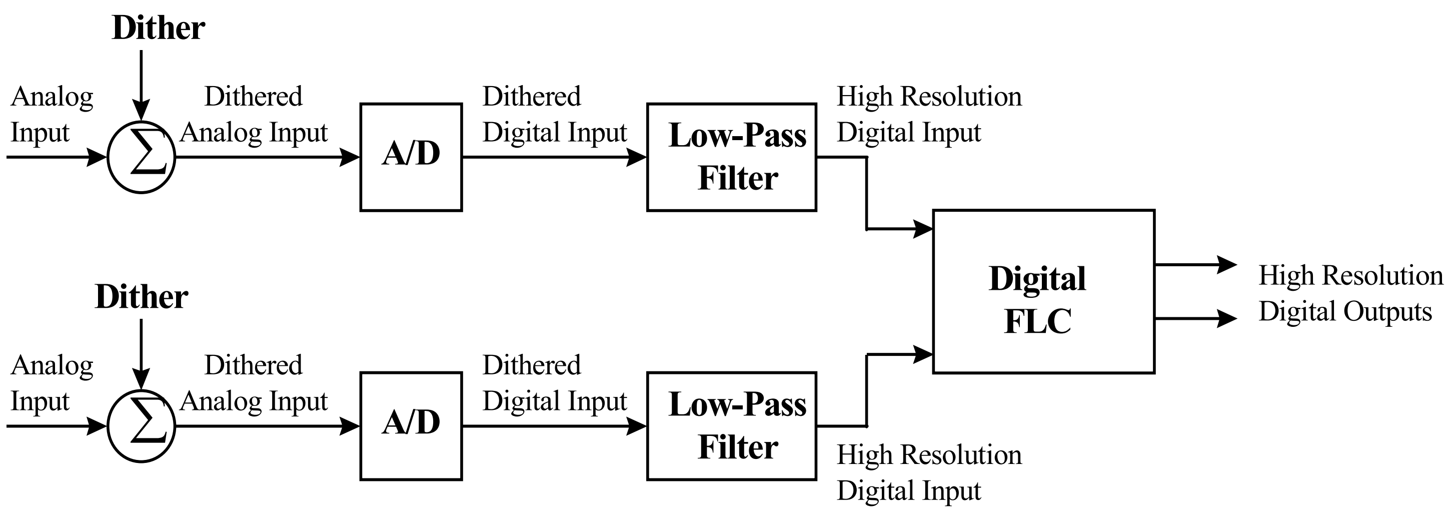


Simulink diagram of a digital FLC for truck backing-up

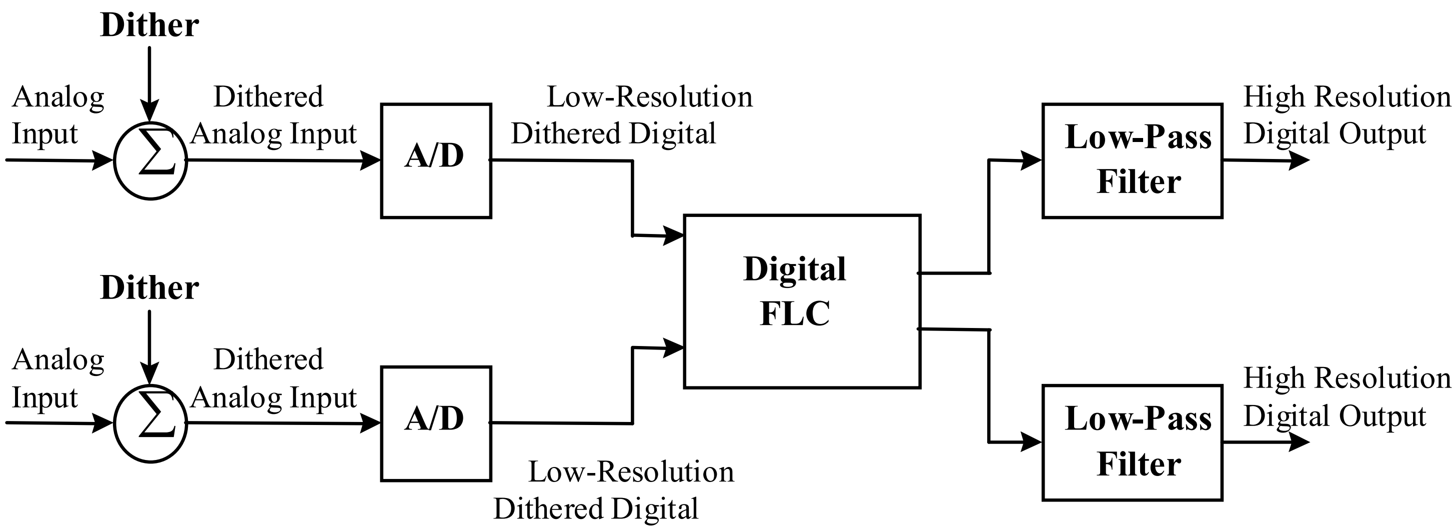




Time diagram of digital FLC's output θ during a docking experiment when the input variables, φ and x are analog and respectively quantizied with a 4-bit bit resolution



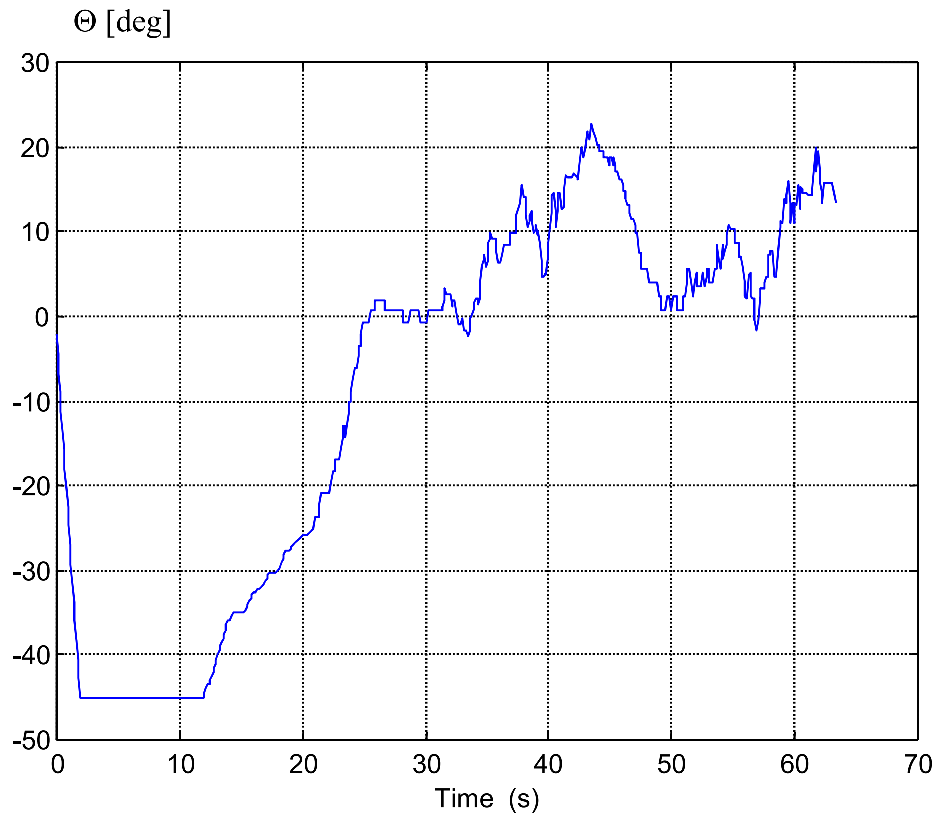
Dithered digital FLC architecture with low-pass filters placed immediately after the input A/D converters

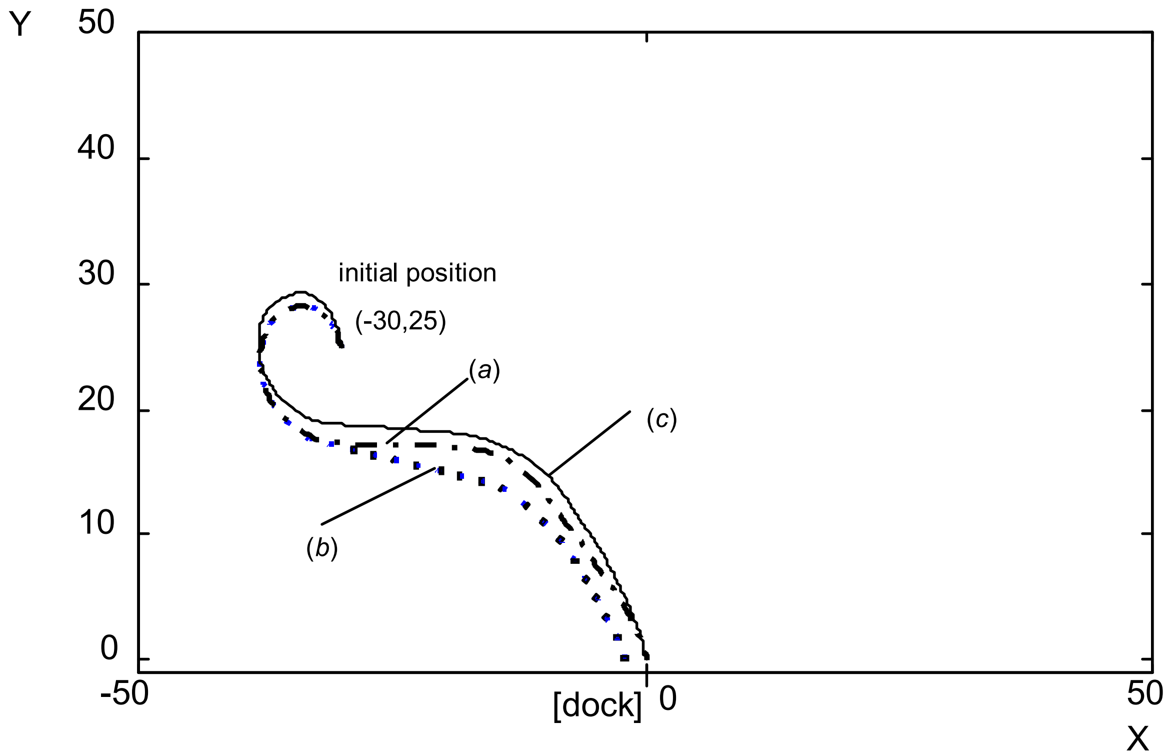


Dithered digital FLC architecture with low-pass filters placed at the FLC's outputs

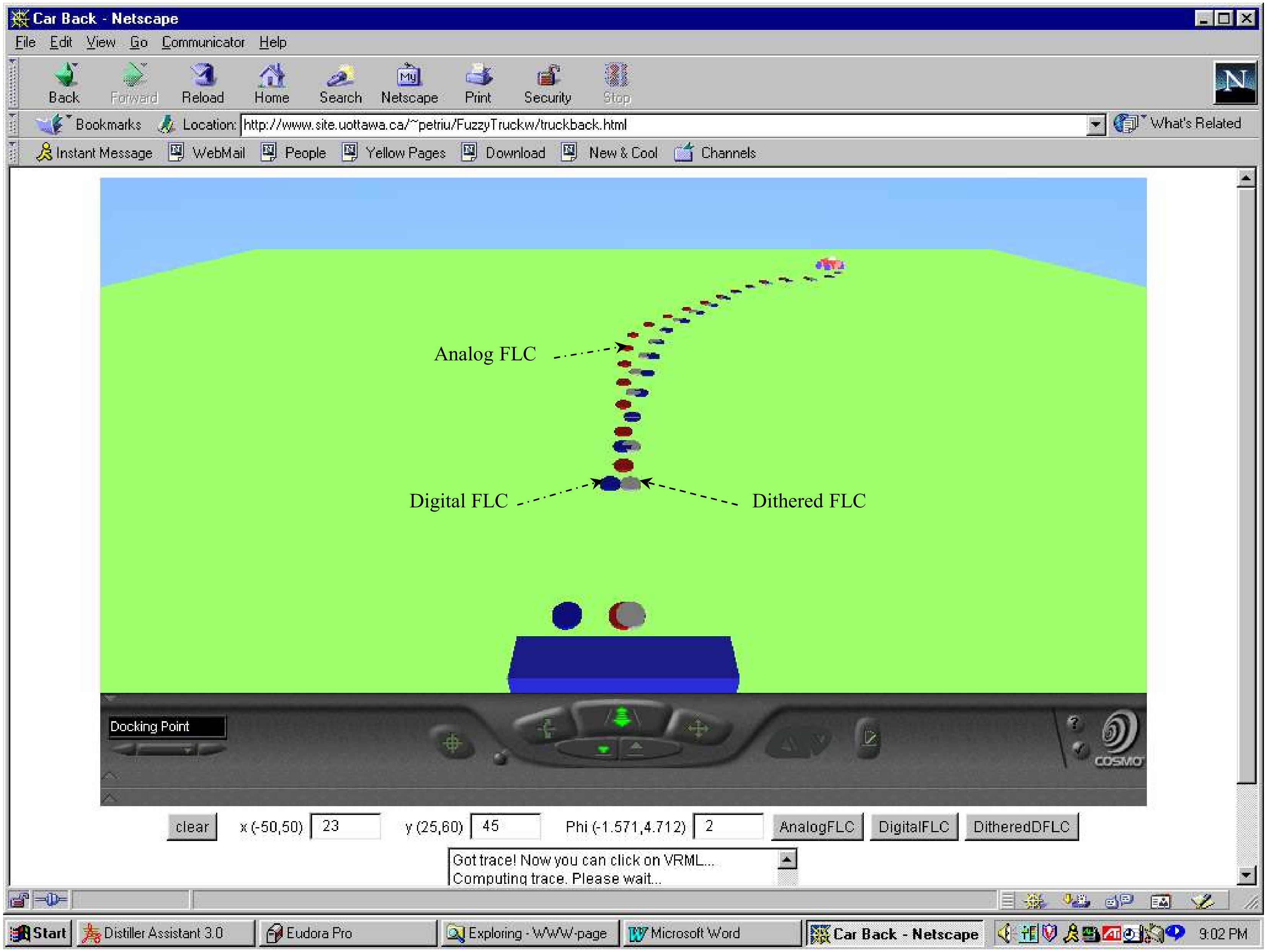
It offers a better performance than the previous one because a final low-pass filter can also smooth the non-linearity caused by the min-max composition rules of the FLC.

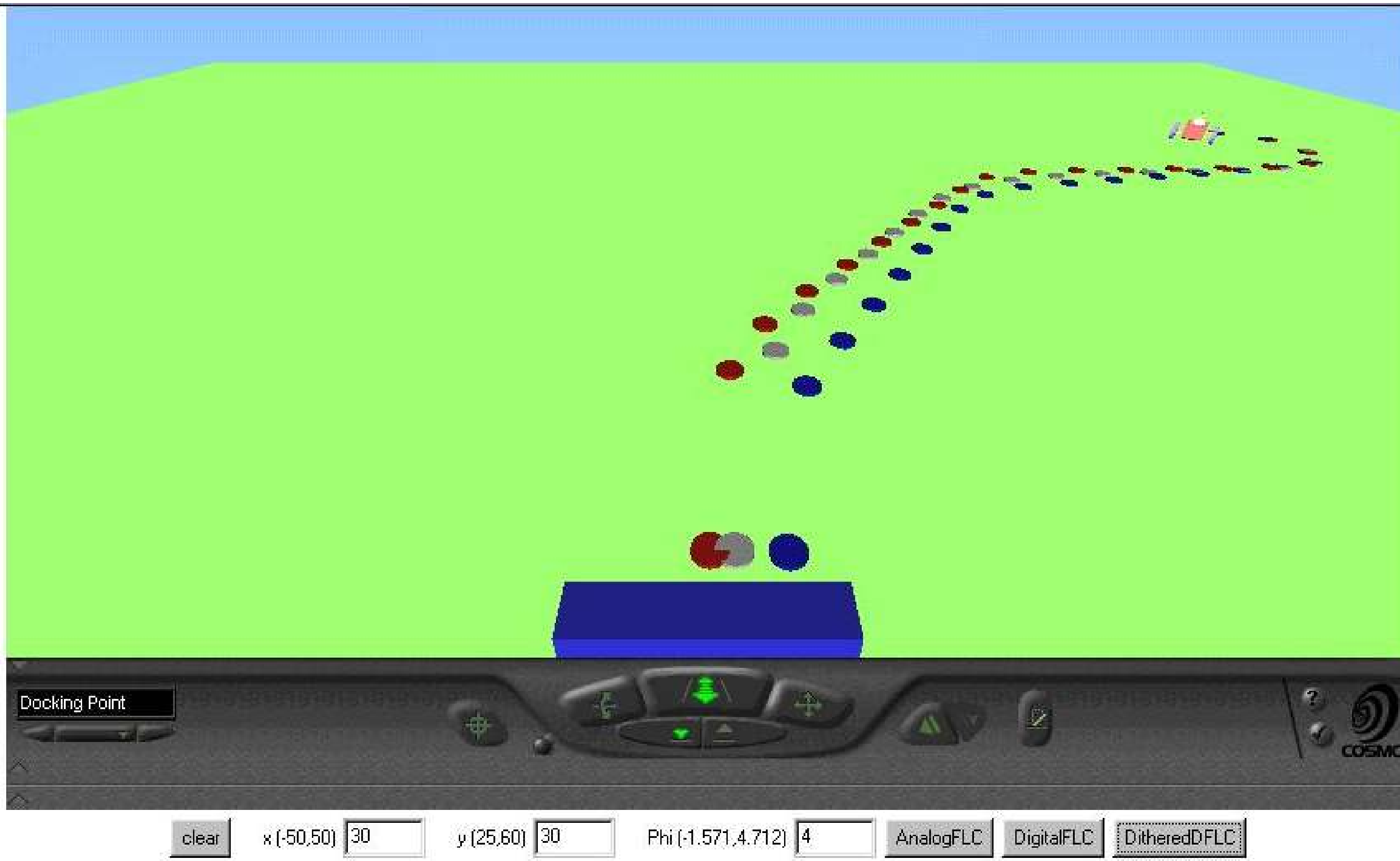
Time diagram of dithered digital FLC's output θ during a docking experiment when 4-bit A/D converters are used to quantize the dithered inputs and the low-pass filter is placed at the FLC's output





Truck trails for different FLC architectures: (a) analog ; (b) digital without dithering; (c) digital with uniform dithering and 20-unit moving average filter


Car Back - Netscape
File Edit View Go Communicator Help
Back Forward Reload Home Search Netscape Print Security Stop
Bookmarks Location: http://www.site.uottawa.ca/~petriu/FuzzyTruckw/truckback.html
What's Related
Instant Message WebMail People Yellow Pages Download New & Cool Channels
Analog FLC
Digital FLC
Dithered FLC
Docking Point
COSMO
clear
x (-50,50) 23
y (25,60) 45
Phi (-1.571,4.712) 2
AnalogFLC DigitalFLC DitheredDFLC
Got trace! Now you can click on VRML...
Computing trace. Please wait...
Start Distiller Assistant 3.0 Eudora Pro Exploring - WWW-page Microsoft Word Car Back - Netscape
9:02 PM


Docking Point
COSMO
clear
x (-50,50)
30
y (25,60)
30
Phi (-1.571,4.712)
4
AnalogFLC
DigitalFLC
DitheredDFLC

Conclusions

- A low resolution of the input data in a digital FLC results in a low resolution of the controller's characteristics.

- Dithering can significantly improve the resolution of a digital FLC beyond the initial resolution of the A/D converters used for the input data.