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ELG3175 Introduction to Communication Systems

VSB and Introduction to Angle Modulation

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Motivation

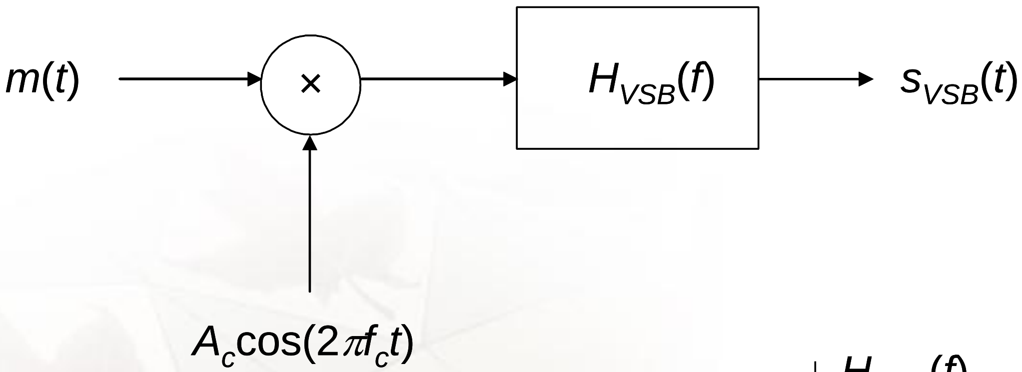
- For wideband information signals, SSB is difficult to implement.
- For frequency discrimination, the filter must have a sharp cutoff near the frequency f_c so as to be able to eliminate one band without distorting the other.
- When we use phase discrimination, we require Hilbert transformers which are difficult to implement if the signal $m(t)$ has a large bandwidth.

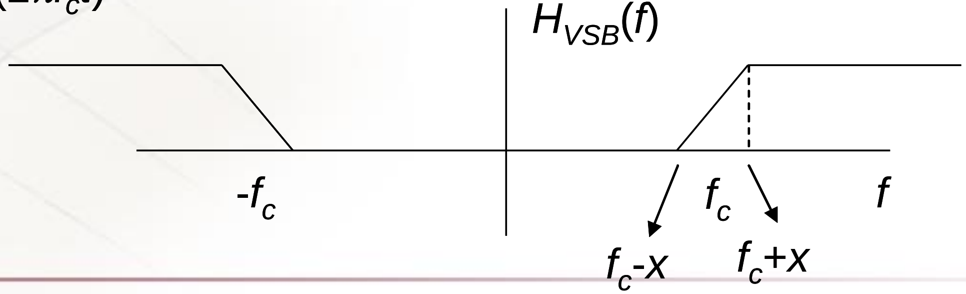
VSB Modulation

- When SSB is difficult to implement, we use vestigial sideband (VSB) modulation.
- VSB is implemented by frequency discrimination but the filtering process does not completely eliminate the unwanted band.
- In fact, some of the desired band is also partially filtered out.

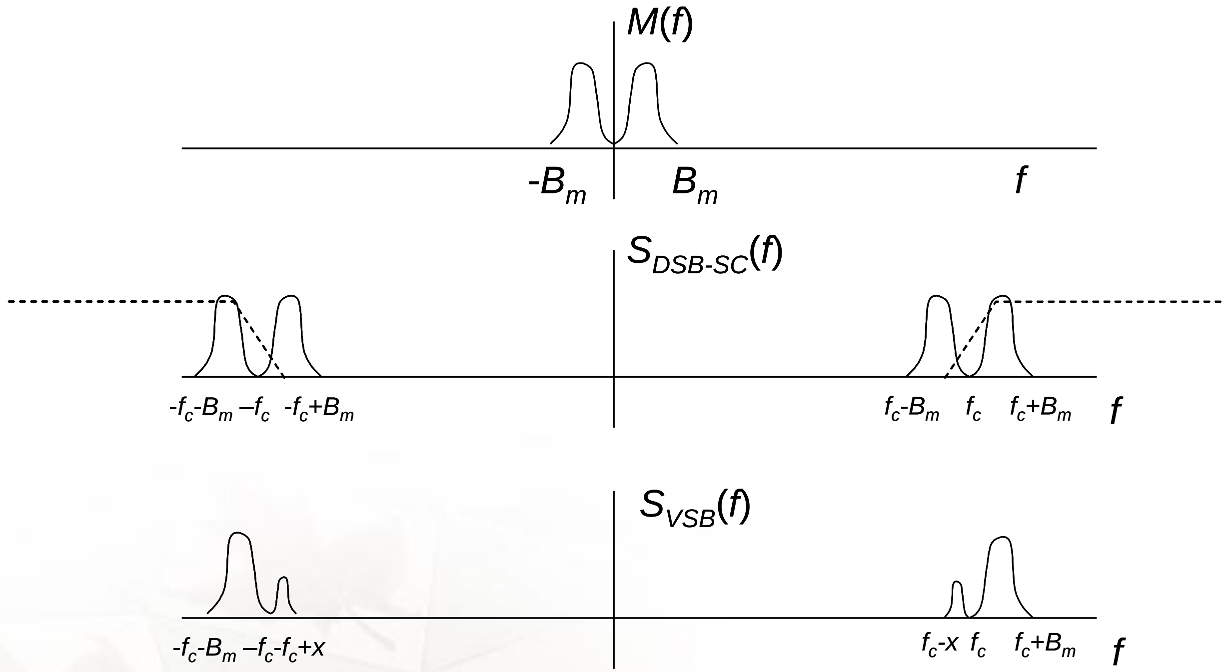
VSB Modulator

- A VSB modulator and its filter's frequency response are shown below.
- The response of the VSB filter is denoted as $H_{VSB}(f)$.
- We notice a transition band around the frequency f_c.





Spectrum of a VSB signal



- In the example given, we consider a system which retains the upper sideband as the principal band and the lower sideband contributes the vestigial sideband part. However, either sideband can be used to form the principal band in practice.

- In the example given, the filter has a gain of K for all frequencies $|f|>f_c+x$ (passband).
- For the frequencies $f_c < |f| < f_c+x$, which reside in the principal band, the gain is less than K, so some loss compared to the passband occurs.
- For the frequencies $f_c-x < |f| < f_c$, the filter's gain is not 0, therefore some of the other sideband's frequency components are passed by the filter and $s_{VSB}(t)$ has a vestigial sideband.
- The bandwidth of $s_{VSB}(t) = B_m+x$. Generally, since we are trying to reduce the bandwidth compared to DSB-SC, x is smaller than B_m.

$S_{VSB}(f)$

$$
\begin{aligned}
S_{VSB}(f) &= S_{DSB-SC}(f)H_{VSB}(f) \\
&= \frac{A_c}{2}M(f-f_c)H_{VSB}(f)+\frac{A_c}{2}M(f+f_c)H_{VSB}(f) \\
&= \frac{A_c}{4}M_+(f-f_c)H^+_{VSB}(f)+\frac{A_c}{4}M_-(f-f_c)H^+_{VSB}(f) \\
&\quad +\frac{A_c}{4}M_+(f+f_c)H^-_{VSB}(f)+\frac{A_c}{4}M_-(f+f_c)H^-_{VSB}(f)
\end{aligned}
$$

where

$$
H^+_{VSB}(f)=\begin{cases} H_{VSB}(f) & f>0 \\ 0 & f<0 \end{cases} \qquad \text{et} \qquad H^-_{VSB}(f)=\begin{cases} 0 & f>0 \\ H_{VSB}(f) & f<0 \end{cases}
$$

We note that $H^-_{VSB}(f)=H^{+*}_{VSB}(-f)$ due to Hermetian symmetry in the frequency response of real systems.

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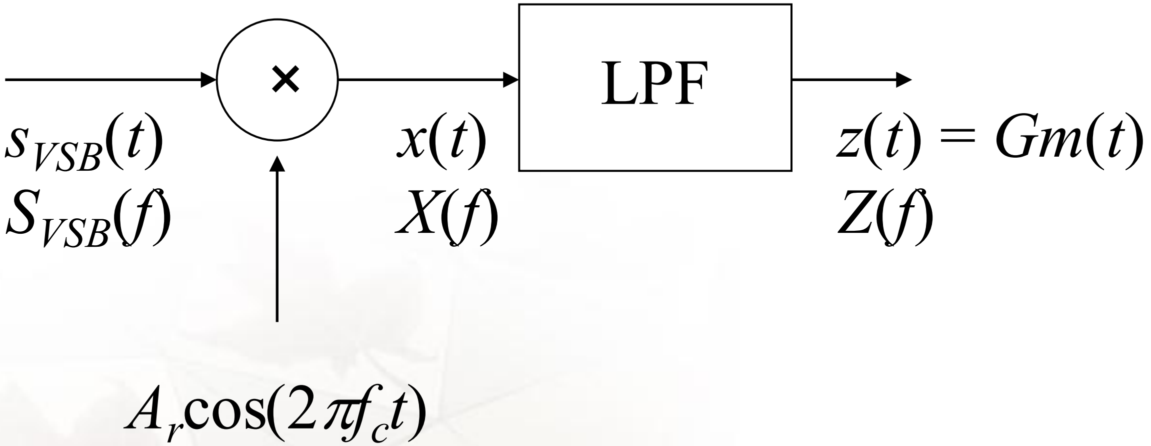
Demodulation of VSB

- We use the same demodulator as DSB-SC



- If we want $z(t) = Gm(t)$, then we need to impose a constraint on the frequency response of the modulator's filter $H_{VSB}(f)$.

$X(f)$

$$
\begin{aligned}
X(f) &= \frac{A_r}{2} S_{VSB}(f - f_c) + \frac{A_r}{2} S_{VSB}(f + f_c) \\
&= \frac{A_c A_r}{8} M_+(f - 2f_c) H^+_{VSB}(f - f_c) + \frac{A_c A_r}{8} M_-(f - 2f_c) H^+_{VSB}(f - f_c) \\
&\quad + \frac{A_c A_r}{8} M_+(f) H^-_{VSB}(f - f_c) + \frac{A_c A_r}{8} M_-(f) H^-_{VSB}(f - f_c) \\
&\quad + \frac{A_c A_r}{8} M_+(f) H^+_{VSB}(f + f_c) + \frac{A_c A_r}{8} M_-(f) H^+_{VSB}(f + f_c) \\
&\quad + \frac{A_c A_r}{8} M_+(f + 2f_c) H^-_{VSB}(f + f_c) + \frac{A_c A_r}{8} M_-(f + 2f_c) H^-_{VSB}(f + f_c)
\end{aligned}
$$

Baseband

$Z(f)$

$$Z(f) = \frac{A_c A_r}{8} M_+(f) H_{VSB}^-(f - f_c) + \frac{A_c A_r}{8} M_-(f) H_{VSB}^-(f - f_c) + \frac{A_c A_r}{8} M_+(f) H_{VSB}^+(f + f_c) + \frac{A_c A_r}{8} M_-(f) H_{VSB}^+(f + f_c)$$

$$Z(f) = \frac{A_c A_r}{8} M_+(f)(H_{VSB}^-(f - f_c) + H_{VSB}^+(f + f_c)) + \frac{A_c A_r}{8} M_-(f)(H_{VSB}^-(f - f_c) + H_{VSB}^+(f + f_c))$$

We want $Z(f) = GM(f)$, where G is a constant. If we ensure that

$$H_{VSB}^-(f - f_c) + H_{VSB}^+(f + f_c) = K$$

$$Z(f) = \frac{A_c A_r K}{8} M_+(f) + \frac{A_c A_r K}{8} M_-(f) = \frac{A_c A_r K}{4} M(f)$$

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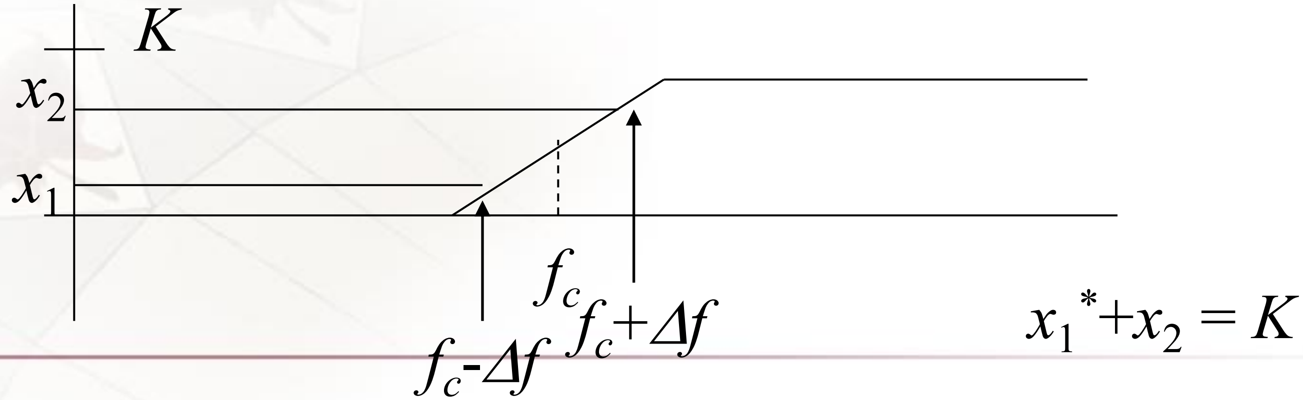
- Therefore $z(t) = (A_c A_r K/4)m(t)$.
- Let us replace $H^{-}_{VSB}(f)$ by $H^{+*}_{VSB}(-f)$ and f by Δf and we get

$$H^{+*}_{VSB}(f_c - \Delta f) + H^{+}_{VSB}(f_c + \Delta f) = K \qquad (***)$$

- (This criteria only need be true over the frequency range of the VSB signal).



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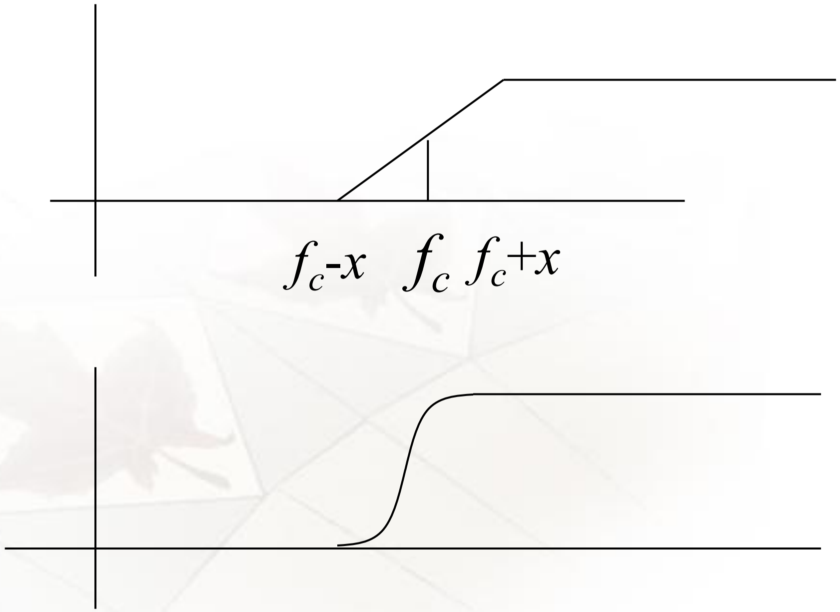
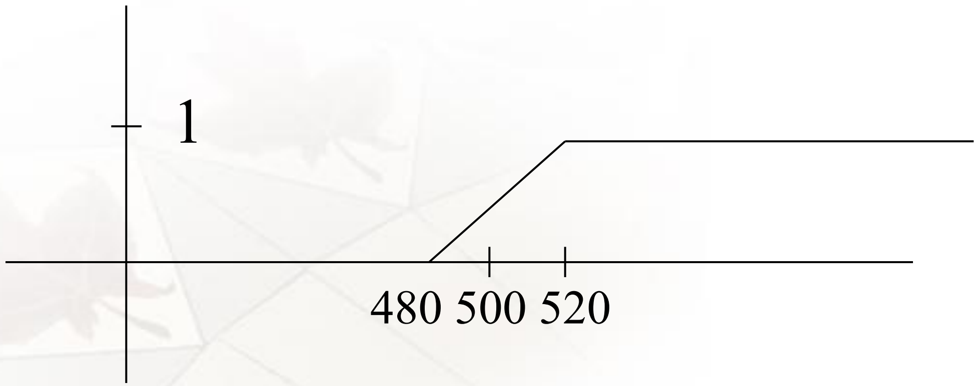
Possible USB filters

- Filters with linear transition bands
- Raised cosine filters



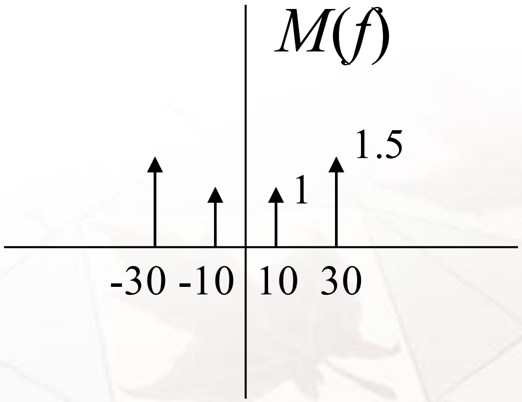
Example

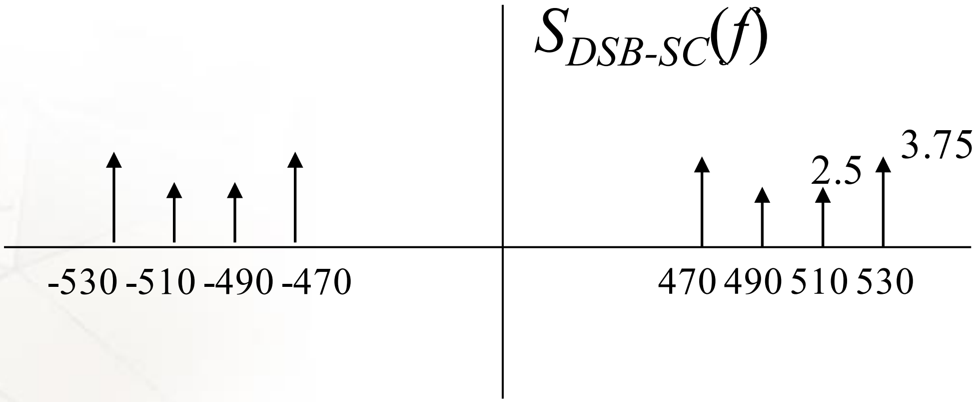
- The signal $m(t) = 2\cos(2\pi 10t)+3\cos(2\pi 30t)$. We wish to transmit this signal using VSB with carrier $c(t) = 5\cos(2\pi 500t)$. The VSB filter's response is shown below. Find $s_{VSB}(t)$ as well as its bandwidth.



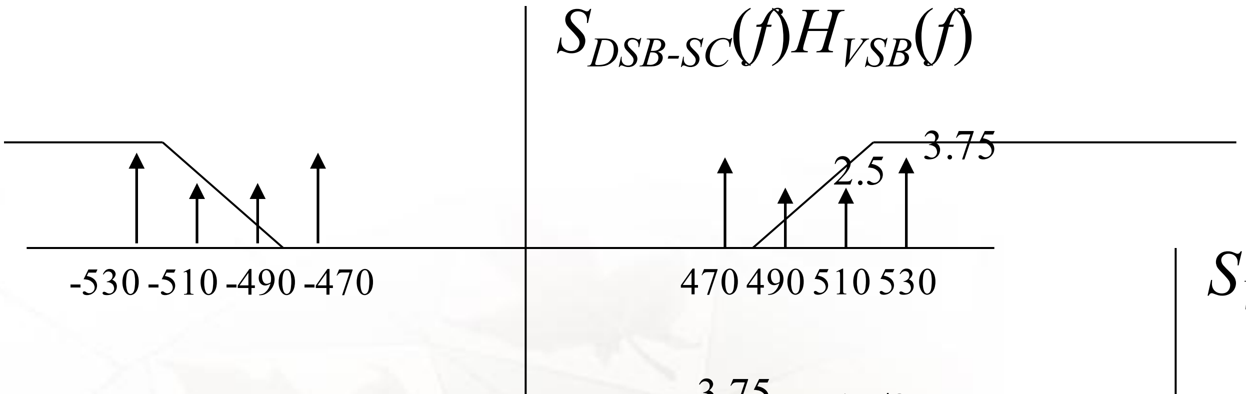
Solution

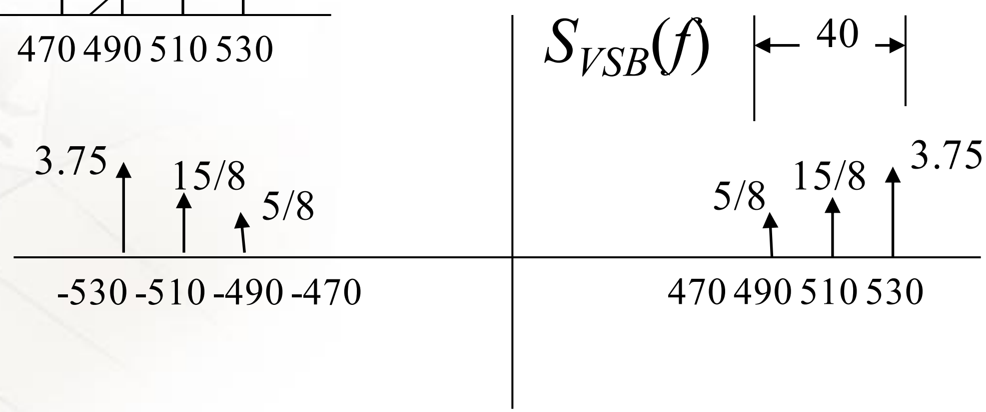
- Since VSB uses frequency discrimination, it is probably best to work in the frequency domain.
- Let us find $M(f)$ and $S_{DSB\text{-}SC}(f)$.





- Next we find $S_{VSB}(f) = S_{DSB\text{-}SC}(f)H_{VSB}(f)$.





$H_{VSB}(470)=0$
$H_{VSB}(490)=1/4$
$H_{VSB}(510)=3/4$
$H_{VSB}(530)=1$

$$s_{VSB}(t) = 7.5\cos(2\pi 530t)+3.75\cos(2\pi 510t)+1.25\cos(2\pi 490t)$$

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Example 2

- Show that we can demodulate $s_{VSB}(t)$ of the previous example to obtain $Gm(t)$.
- $s_{VSB}(t)\cos(2\pi500t) = 7.5\cos(2\pi530t)\cos(2\pi500t) + 3.75\cos(2\pi510t)\cos(2\pi500t) + 1.25\cos(2\pi490t)\cos(2\pi500t) = 3.75\cos(2\pi30t) + 3.75\cos(2\pi1030t) + 1.875\cos(2\pi10t) + 1.875\cos(2\pi1010t) + 0.625\cos(2\pi10t) + 0.625\cos(2\pi990t)$.
- After lowpass filtering $z(t) = 3.75\cos(2\pi30t) + 1.875\cos(2\pi10t) + 0.625\cos(2\pi10t) = 3.75\cos(2\pi30t) + 2.5\cos(2\pi10t) = 1.25m(t)$.

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Introduction to Angle Modulation

- In angle modulation, the amplitude of the modulated signal remains fixed while the information is carried by the angle of the carrier.
- The process that transforms a message signal into an angle modulated signal is a nonlinear one.
- This makes analysis of these signals more difficult.
- However, their modulation and demodulation are rather simple to implement.

The angle of the carrier

- Let $\theta_i(t)$ represent the instantaneous angle of the carrier.
- We express an angle modulated signal by:

$$s(t) = A_c \cos\left(\theta_i(t)\right)$$

- where A_c is the carrier amplitude.

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Instantaneous frequency

- One cycle occurs when $\theta_i(t)$ changes by 2π radians, therefore the average frequency of $s(t)$ on the interval t to $t+\Delta t$ is:

$$f_{\Delta t} = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

- Therefore the instantaneous frequency is found in the limit as Δt tends towards 0.

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

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Phase modulation

- There are two angle modulation techniques.
  - Phase modulation (PM)
  - Frequency modulation (FM)
- In PM, the phase of the carrier is a linear function of the message signal, $m(t)$. Therefore $s_{PM}(t)$ is:

$$s_{PM}(t) = A_c \cos\left(2\pi f_c t + k_p m(t) + \phi_c\right)$$

- where k_p is the phase sensitivity and ϕ_c is the phase of the unmodulated carrier.
- To simplify expressions, we will assume that $\phi_c = 0$. Therefore the angle of a PM signal is given by $\theta_i(t) = 2\pi f_c t + k_p m(t)$.

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FM

- For FM, the instantaneous frequency is a linear function of the message:

$$f_i(t) = f_c + k_f m(t)$$

- where k_f is the frequency sensitivity.

$$\theta_i(t) = 2\pi \int_{-\infty}^{t} f_i(\tau)d\tau = 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]$$

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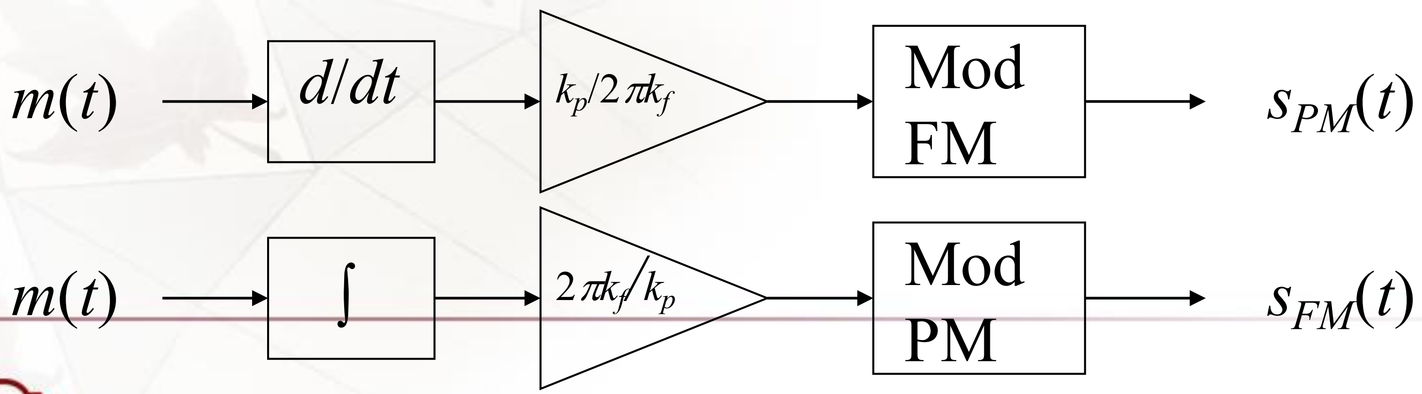
Instantaneous frequency of a PM signal / Instantaneous phase of an FM signal

- From $s_{PM}(t)$, we find

$$f_i(t)_{PM} = f_c + \frac{k_p}{2\pi}\frac{dm(t)}{dt}$$

- From $s_{FM}(t)$, we find

$$\phi_i(t)_{FM} = 2\pi k_f \int_0^t m(\tau)d\tau$$



Example

- Find $s_{FM}(t)$ and $s_{PM}(t)$ if $m(t) = A\cos(2\pi f_m t)$.
  - SOLUTION

$$s_{PM}(t) = A_c \cos\left[2\pi f_c t + Ak_p \cos\left(2\pi f_m t\right)\right]$$

$$\int_{-\infty}^{t} A\cos\left(2\pi f_m \tau\right) d\tau = \frac{A}{2\pi f_m}\sin\left(2\pi f_m t\right)$$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{Ak_f}{f_m}\sin\left(2\pi f_m t\right)\right]$$

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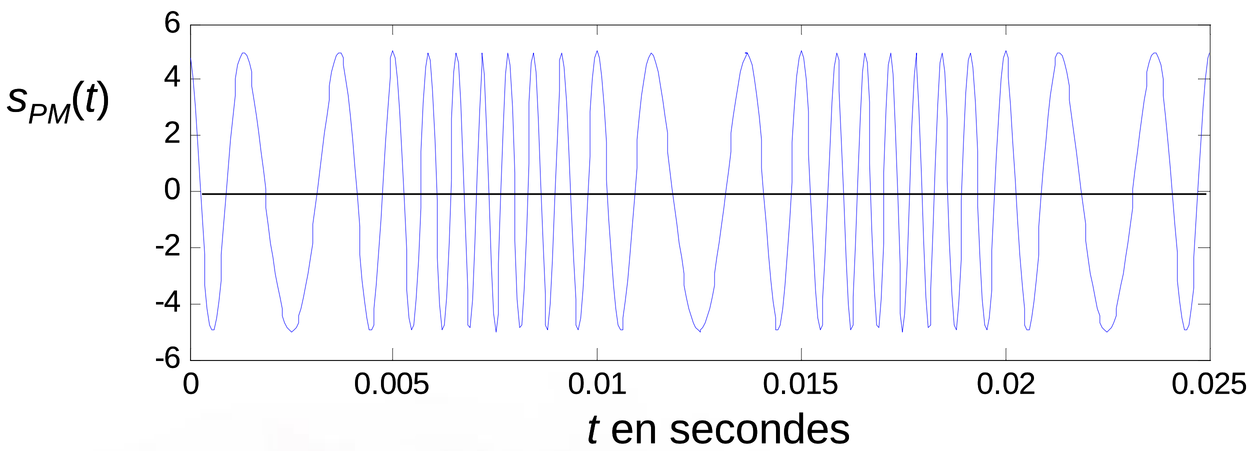
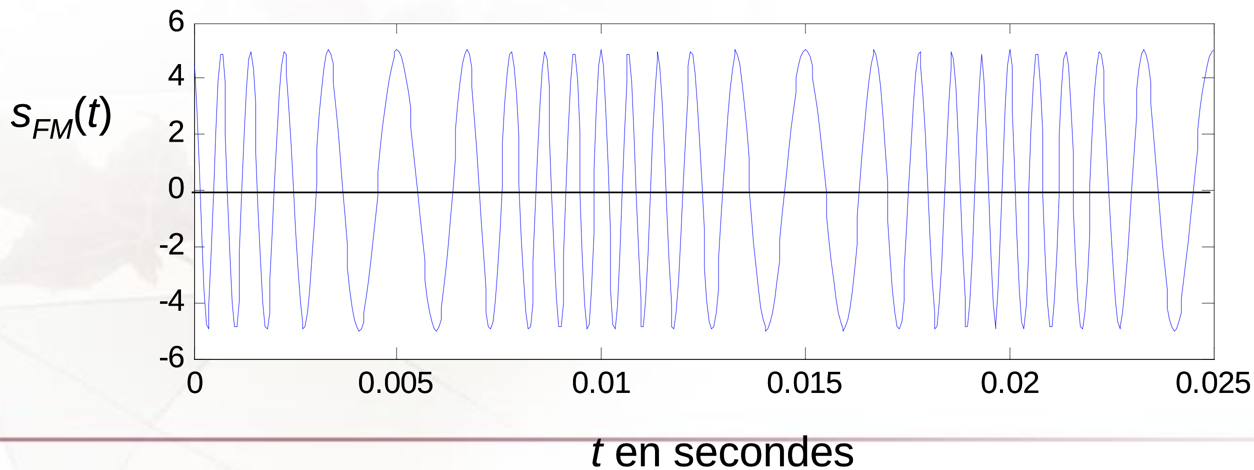
- The PM and FM of the example are shown here for $A_c = 5$, $A = 1$, $f_c = 1$ kHz, $f_m = 100$ Hz, $k_p = 2\pi$ rads/V and $k_f = 500$ Hz/V.





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