ELG3175 Introduction to Communication Systems

Quadrature and Single Sideband AM

Quadrature Amplitude Modulation (QAM)

- Spectral efficiency refers to the amount of information that we can transmit per unit bandwidth
- DSB-SC transmita a signal with bandwidth B_m on a bandpass bandwidth of $2B_m$.
- QAM transmits two signals of bandwidth B_m on a bandpass bandwidth of $2B_m$.
- Therefore it uses bandwidth twice as efficiently.

QAM 2

- A QAM signal is in the form:

$$s_{QAM}(t) = A_c m_1(t)\cos 2\pi f_c t + A_c m_2(t)\sin 2\pi f_c t$$

- Where
  - A_c is the carrier amplitude
  - $m_1(t)$ and $m_2(t)$ are independent information signals, both with bandwidth B_m.
  - f_c is the carrier frequency and $f_c >> B_m$.

The spectrum of a QAM signal

- Le spectre d'un signal QAM est :

$$S_{QAM}(f) = \frac{A_c}{2} M_1(f - f_c) + \frac{A_c}{2} M_1(f + f_c) + \frac{A_c}{2j} M_2(f - f_c) - \frac{A_c}{2j} M_2(f + f_c)$$

Demodulation of $m_1(t)$

- Let us multiply $s_{QAM}(t)$ by $A_r\cos 2\pi f_c t$, which gives us:

$$
\begin{aligned}
A_r s_{QAM}(t)\cos 2\pi f_c t &= A_c A_r m_1(t)\cos^2 2\pi f_c t + A_c A_r m_2(t)\sin 2\pi f_c t\cos 2\pi f_c t \\
&= \underbrace{\frac{A_c A_r}{2} m_1(t)}_{\text{basebandsignal}} + \underbrace{\frac{A_c A_r}{2} m_1(t)\cos 4\pi f_c t + \frac{A_c A_r}{2} m_2(t)\sin 4\pi f_c t}_{\text{bandpasssignal centredat } f=2f_c}
\end{aligned}
$$

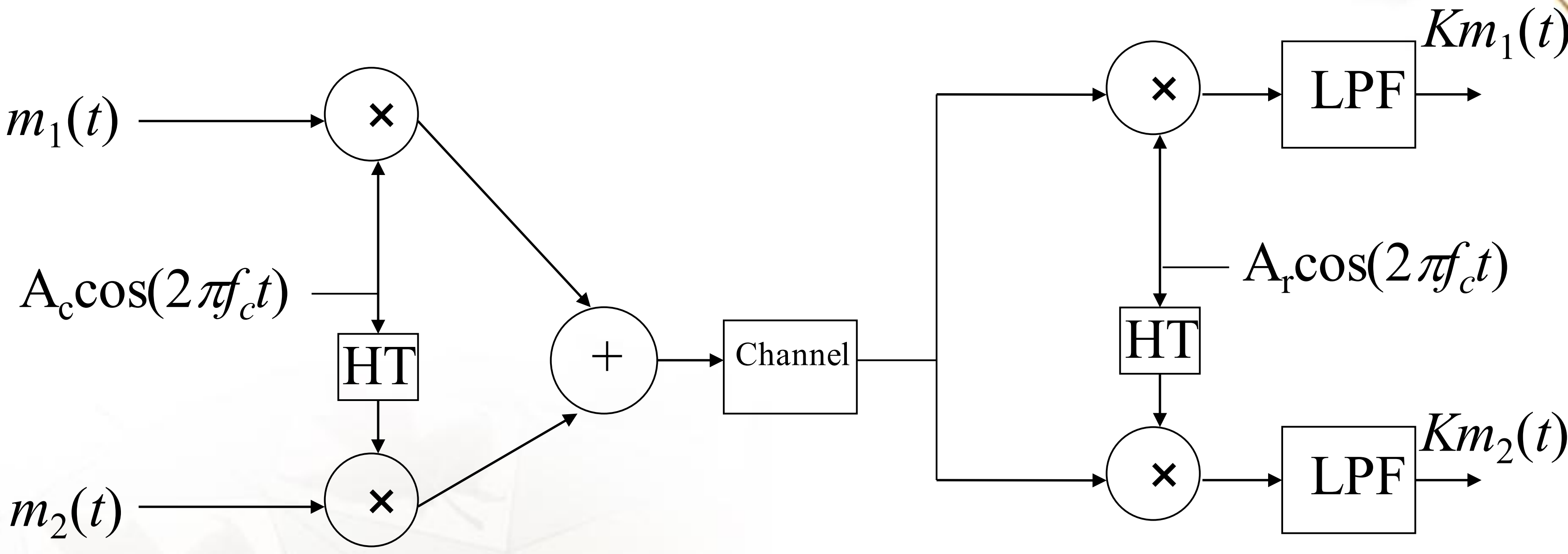
$$(\cos A \sin A = 0.5\sin 2A).$$

Demodulation of $m_2(t)$

- Similarly, if we multiply $s_{QAM}(t)$ by $A_r \sin 2\pi f_c t$, we get:

$$
\begin{aligned}
A_r s_{QAM}(t)\sin 2\pi f_c t &= A_c A_r m_2(t)\sin^2 2\pi f_c t + A_c A_r m_1(t)\cos 2\pi f_c t \sin 2\pi f_c t \\
&= \underbrace{\frac{A_c A_r}{2} m_2(t)}_{\text{baseband signal}} \underbrace{- \frac{A_c A_r}{2} m_2(t)\cos 4\pi f_c t + \frac{A_c A_r}{2} m_1(t)\sin 4\pi f_c t}_{\text{bandpass signal centred at } f=2f_c}
\end{aligned}
$$

QAM System

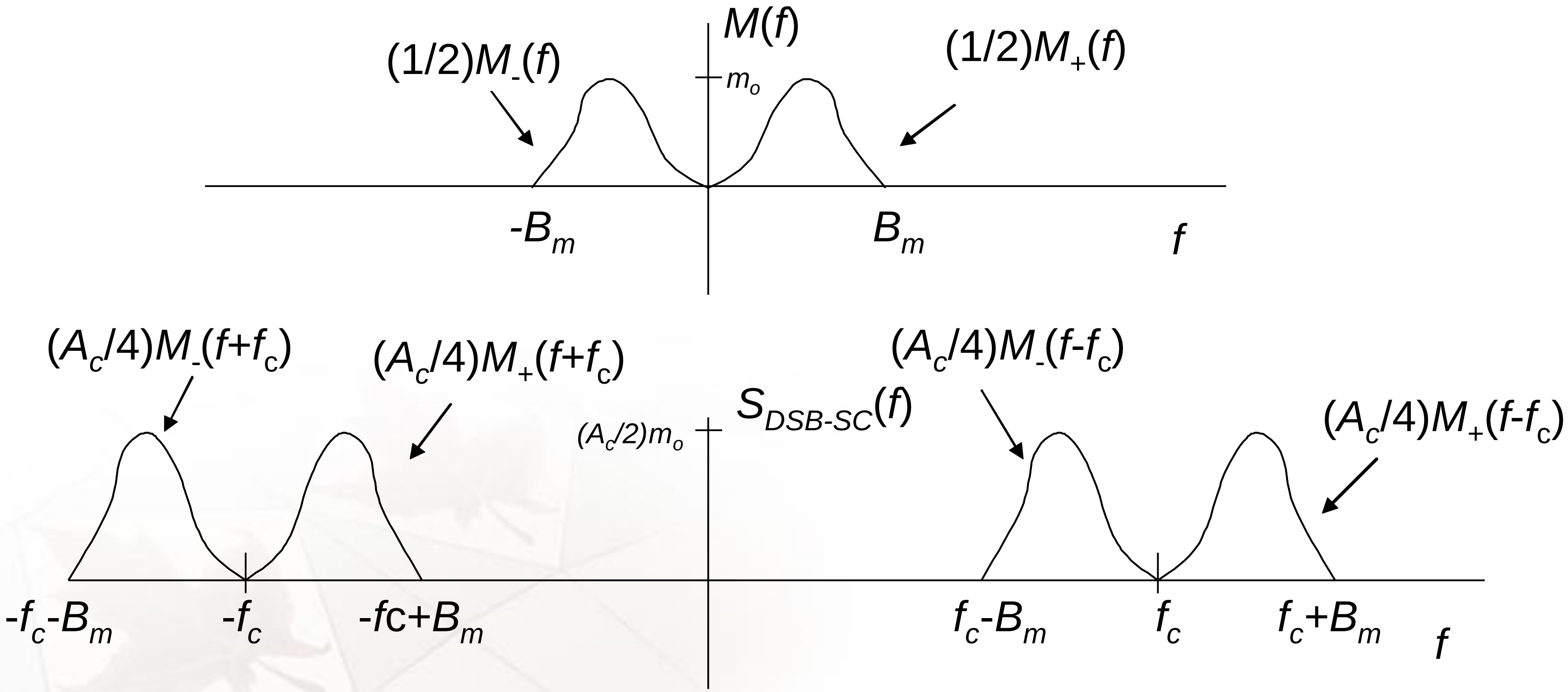


$m_1(t)$
$A_c\cos(2\pi f_c t)$
HT
$m_2(t)$
+
Channel
$A_r\cos(2\pi f_c t)$
HT
LPF
$Km_1(t)$
LPF
$Km_2(t)$

Advantages and disadvantages

- Can multiplex twice as much information on the same bandwidth as DSB-SC
- Even more sensitive to carrier and phase errors compared to DSB-SC.
  - Crosstalk.

DSB-SC spectrum



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Motivation

- From the previous slide, we see that $S_{DSB\text{-}SC}(f) = (A_c/4)M_+(f-f_c) + (A_c/4)M_-(f-f_c) + (A_c/4)M_+(f+f_c) + (A_c/4)M_-(f+f_c)$.
- The spectrum of a DSB-SC signal has two “copies” of the positive pre-envelope of $m(t)$ and two “copies” of its negative pre-envelope.
- Actually, we would only need one of each to perfectly reconstruct $m(t)$.
- By eliminating one of the sidebands, we obtain single sideband (SSB) AM.

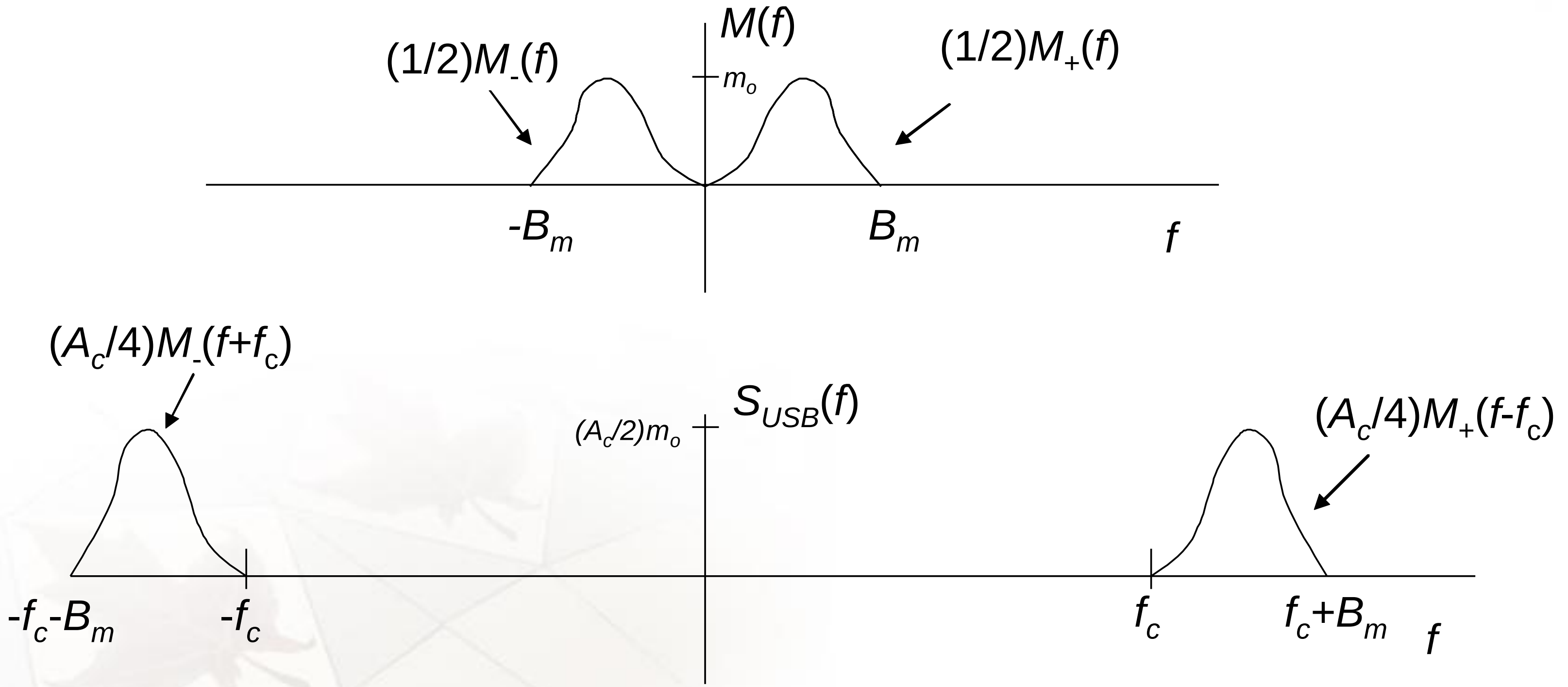
Upper Sideband (USB)

- The upper sideband of a DSB-SC signal is one that has spectrum $S_{USB}(f)$:

$$S_{USB}(f) = \begin{cases} S_{DSB-SC}(f), & |f| > f_c \\ 0, & \text{otherwise} \end{cases}$$

- Compared to a DSB-SC signal that occupies a bandwidth of $2B_m$, the spectrum of a USB signal occupies the frequency range $f_c < |f| < f_c + B_m$, therefore it has half the bandwidth of a DSB-SC signal.

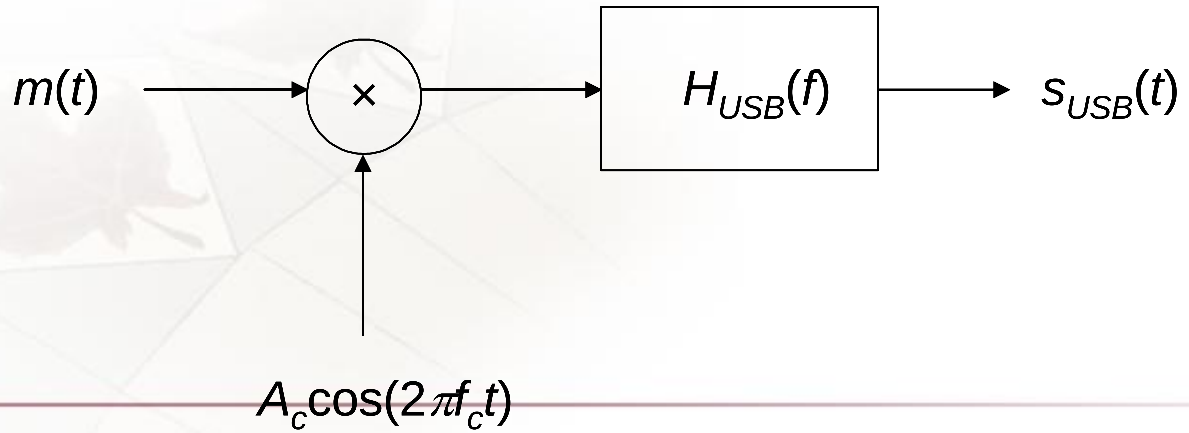
Spectrum of USB signal



USB Modulation using frequency discrimination

- We can produce USB modulation by two methods: Frequency discrimination or phase discrimination.
- For frequency discrimination, we use a high pass filter on a DSB-SC signal.

$$H_{USB}(f) = \begin{cases} 1 & |f| \geq f_c \\ 0 & |f| \leq f_c \end{cases}$$



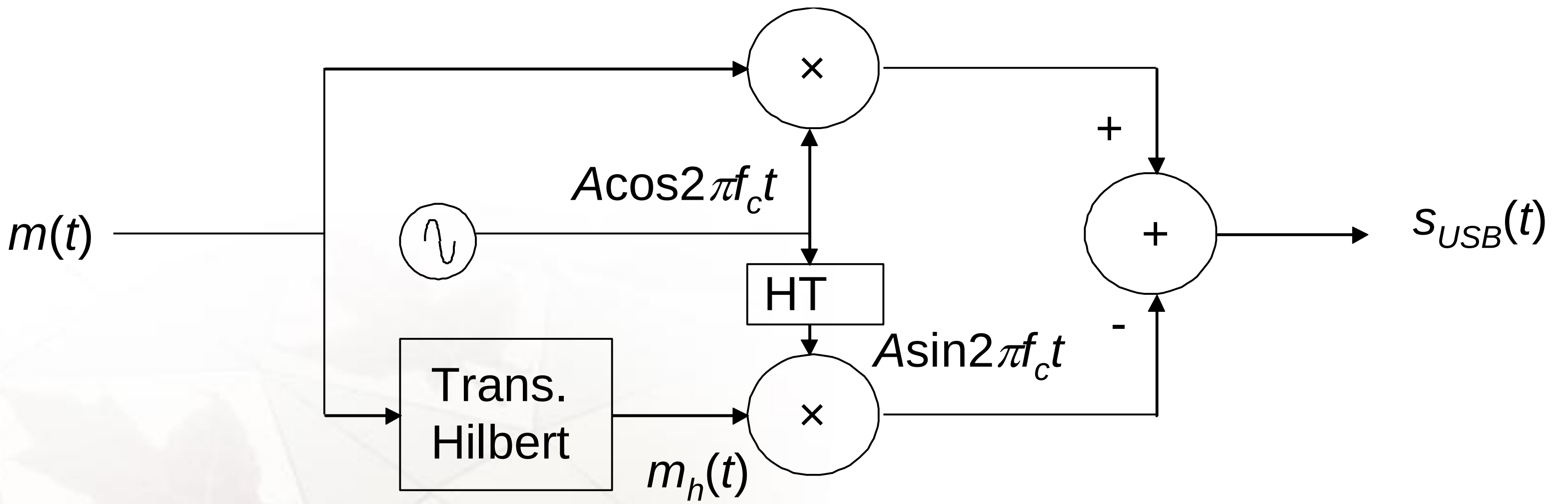
USB Modulation by phase discrimination

- Let us consider the USB signal's spectrum.
- $S_{USB}(f) = (A_c/4)M_+(f-f_c) + (A_c/4)M_-(f+f_c)$.
- Taking the inverse Fourier Transform we get:

$$\begin{aligned} s_{USB}(t) &= \tfrac{A_c}{4} m_+(t) e^{j2\pi f_c t} + \tfrac{A_c}{4} m_-(t) e^{-j2\pi f_c t} \\ &= \tfrac{A_c}{4}(m(t) + jm_h(t))(\cos 2\pi f_c t + j\sin 2\pi f_c t) + \\ &\quad \tfrac{A_c}{4}(m(t) - jm_h(t))(\cos 2\pi f_c t - j\sin 2\pi f_c t) \\ &= \tfrac{A_c}{2}(m(t)\cos 2\pi f_c t - m_h(t)\sin 2\pi f_c t) \end{aligned}$$

$$s_{USB}(t) = Am(t)\cos 2\pi f_c t - Am_h(t)\sin 2\pi f_c t$$

USB Phase Discriminator Modulator

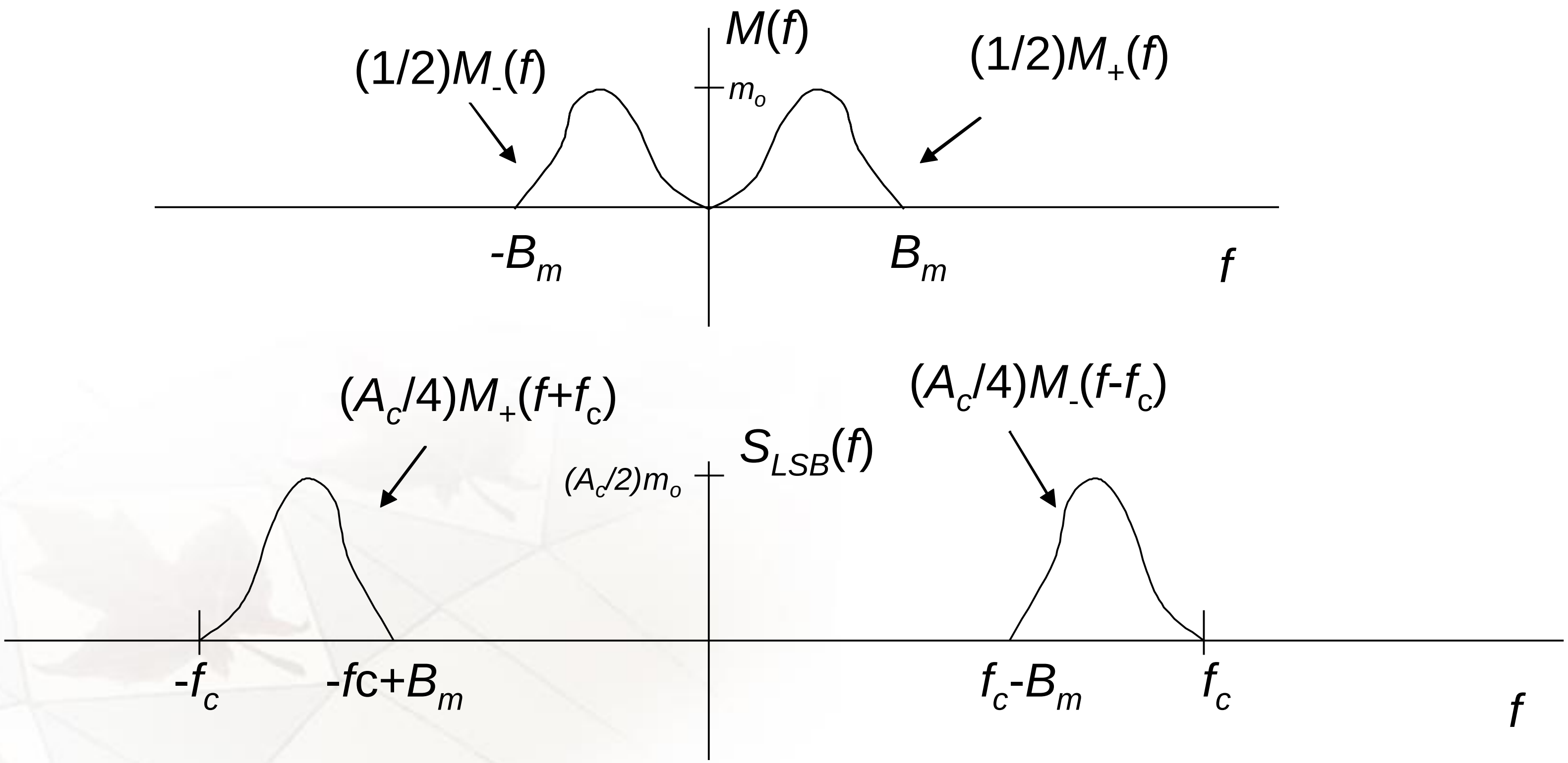


Lower Sideband Modulation (LSB)

- The lower sideband of a DSB-SC signal is the part of the spectrum where $|f|<f_c$.
- Therefore the spectrum of an LSB signal is $S_{LSB}(f)$ which is given by:

$$S_{LSB}(f)=\begin{cases} S_{DSB-SC}(f), & |f|<f_c \\ 0, & \text{otherwise} \end{cases}$$

LSB Spectrum



LSB Modulation by frequency discrimination

- We input the DSB-SC signal to a lowpass filter with frequency response :

$$H_{LSB}(f) = \begin{cases} 1 & |f| \leq f_c \\ 0 & |f| > f_c \end{cases}$$

LSB modulation by phase discrimination

- We can show that $s_{LSB}(t)$ is given by:

$$s_{LSB}(t) = Am(t)\cos 2\pi f_c t + Am_h(t)\sin 2\pi f_c t$$

Examples

- The messge is $m(t) = \cos(2\pi f_m t)$. Find the USB and LSB signals for a carrier amplitude of A and carrier frequency $f_c >> f_m$.
- **Solution (phase discriminator)**
- $s_{USB}(t) = A\cos(2\pi f_m t)\cos(2\pi f_c t) - A\sin(2\pi f_m t)\sin(2\pi f_c t) = (A/2)\cos(2\pi(f_c-f_m)t) + (A/2)\cos(2\pi(f_c+f_m)t) - (A/2)\cos(2\pi(f_c-f_m)t) + (A/2)\cos(2\pi(f_c+f_m)t) = A\cos(2\pi(f_c+f_m)t)$.
- Similarly we can show that $s_{LSB}(t) = A\cos(2\pi(f_c-f_m)t)$.
- **Solution (discfrequency discrmination)**
- $s_{DSB\text{-}SC}(t) = A_c\cos(2\pi f_m t)\cos(2\pi f_c t)$.
- $S_{DSB\text{-}SC}(f) = (A_c/4)\delta(f-f_c-f_m)+(A_c/4)\delta(f+f_c+f_m)+(A_c/4)\delta(f-f_c+f_m)+(A_c/4)\delta(f+f_c-f_m)$.

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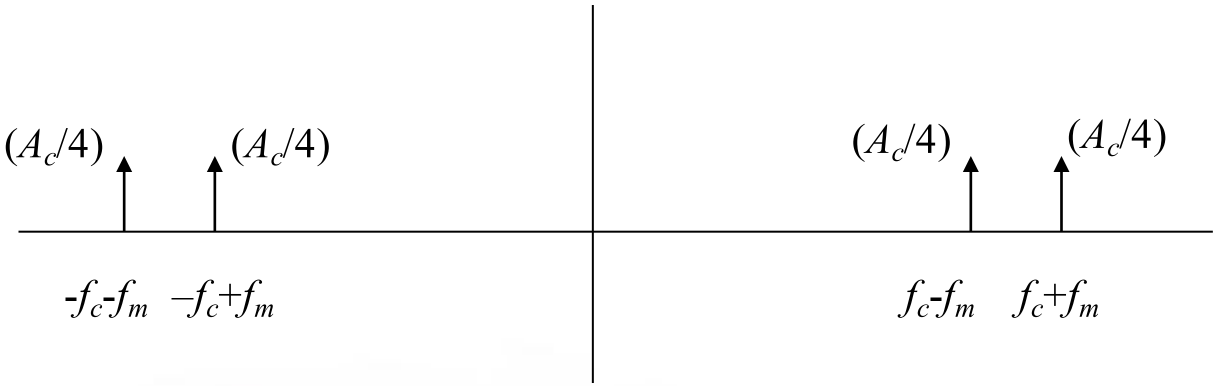

therefore $S_{USB}(f) = (A_c/4)\delta(f-f_c-f_m)+(A_c/4)\delta(f+f_c+f_m)$ and

$s_{USB}(t) = (A_c/2)\cos(2\pi(f_c+f_m)t) = A\cos(2\pi(f_c+f_m)t)$

similarly

$S_{LSB}(f) = (A_c/4)\delta(f-f_c+f_m)+(A_c/4)\delta(f+f_c-f_m)$ and

$s_{LSB}(t) = (A_c/2)\cos(2\pi(f_c+f_m)t) = A\cos(2\pi(f_c+f_m)t)$

Demodulation of SSB

- Same as DSB-SC