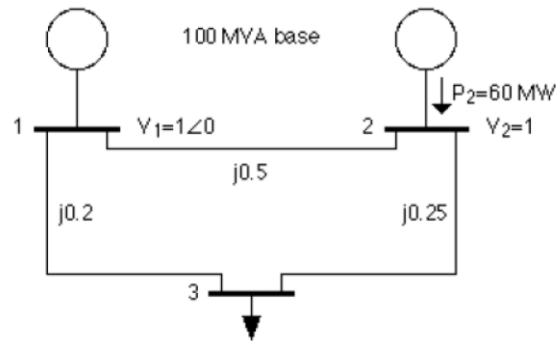


## ELG4125: Lab Assignment on Power Flow

This assignment MUST to be solved and submitted INDIVIDUALLY!



**Figure 1:** Three-bus power system used for load flow calculations. Voltages and line impedances are in per unit. System base is 100 MVA

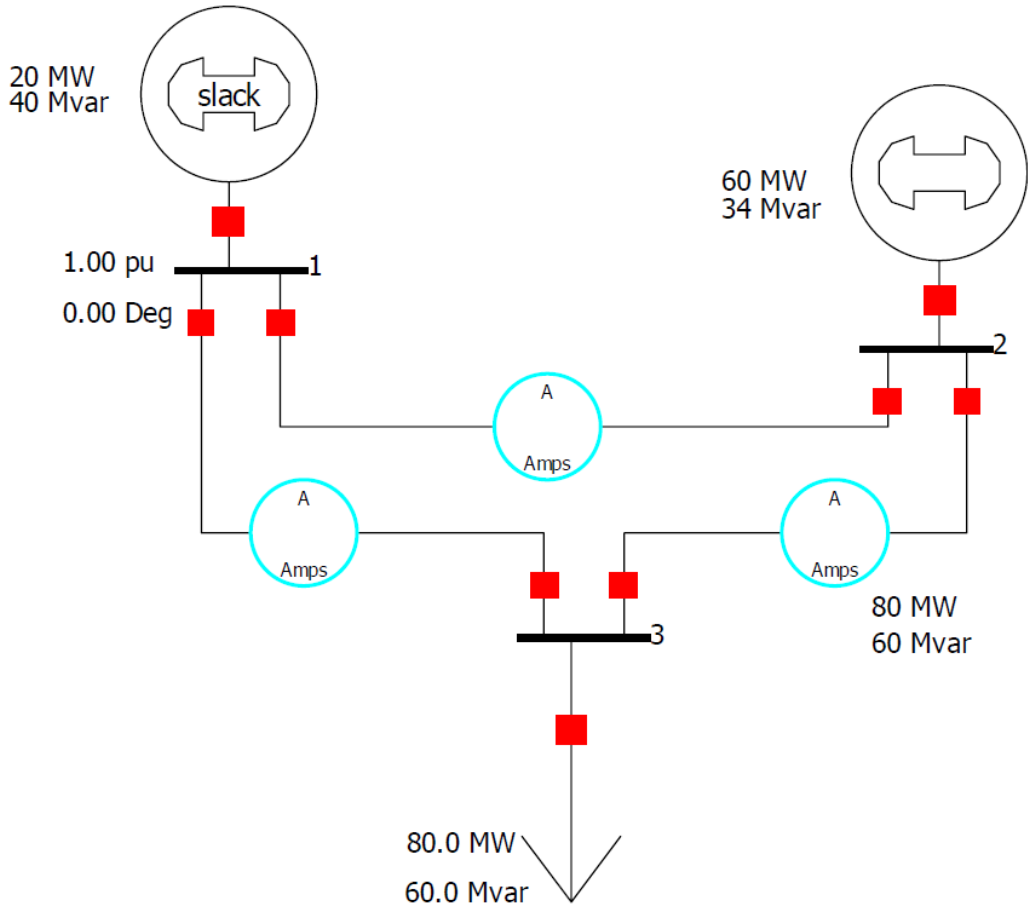
### Identification of Buses!

	Bus	1	2	3
	a)	slack	PV	PQ
	b)	$V_1$ and $\theta_1$ known	$P_2$ known $V_2$ known	$P_3$ and $Q_3$ known
c)	$P_1$ and $Q_1$ unknown	$Q_2$ unknown $\theta_2$ unknown	$V_3$ and $\theta_3$ unknown	

### 1. Start PowerWorld and enter the system in Figure 1 in Edit Mode:

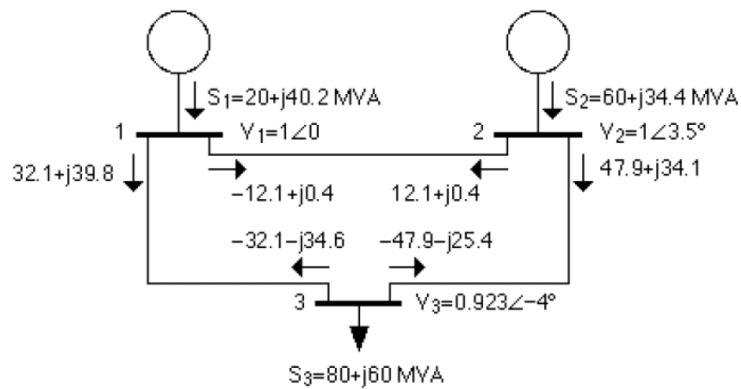
- Insert bus 1. Check that it is used as system slack bus and that the voltage and phase angle are correct. 138 kV default is OK as nominal voltage.
- Insert bus 2 and 3.
- Insert generators at buses 1 and 2 and a load at bus 3. Enter generation and load power. Check that the voltage set point is correct at bus 2.
- Insert the three transmission lines and enter their line reactance in p.u.

(See the procedure in the following graph)



**2. Full load flow solution:**

- Change to Run Mode and push Single Solution to solve the load flow.
- Analyze results (Case Information: Power Flow List, Buses...). Compare with Figure 2.



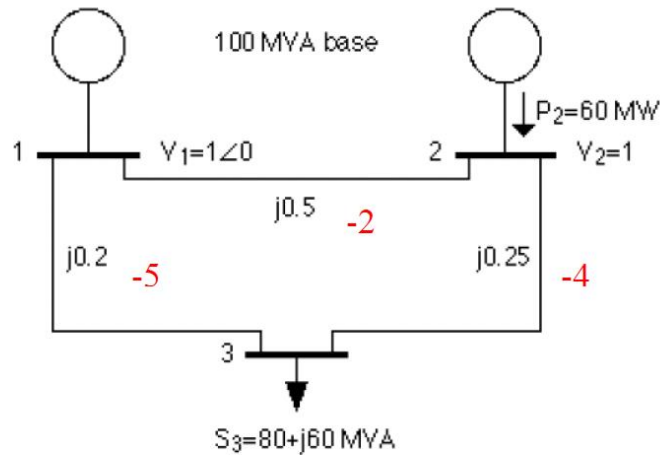
**Figure 2:** Load flow solution and line flows of system in Figure 1. Note that voltage is in p.u. while line flows are in MW and MVAR.

(See the following table)

BUS	1	1	138.0	MW	Mvar	MVA	%	1.0000	0.00	1	1
GENERATOR	1		20.00		40.14R	44.8					
TO	2	2	1	-12.10	0.36	12.1	0				
TO	3	3	1	32.10	39.78	51.1	0				
BUS	2	2	138.0	MW	Mvar	MVA	%	1.0000	3.47	1	1
GENERATOR	1		60.00		34.43R	69.2					
TO	1	1	1	12.10	0.37	12.1	0				
TO	3	3	1	47.90	34.06	58.8	0				
BUS	3	3	138.0	MW	Mvar	MVA	%	0.9227	-3.99	1	1
LOAD	1		80.00		60.00	100.0					
TO	1	1	1	-32.10	-34.55	47.2	0				
TO	2	2	1	-47.90	-25.42	54.2	0				

**3. Matrices: In Run Mode, select Reset to flat start in the Simulation menu.**

- a) Set up the 3x3 per unit bus admittance matrix for the system in Figure 1.



$$Y_{bus} = j \begin{bmatrix} -7 & 2 & 5 \\ 2 & -6 & 4 \\ 5 & 4 & -9 \end{bmatrix} p.u.$$

- b) Select Y bus in the Case information menu.  
c) Compare it to your own solution in “a”.

Number	Name	Bus 1	Bus 2	Bus 3
1	1	0.00 - j7.00	-0.00 + j2.00	-0.00 + j5.00
2	2	-0.00 + j2.00	0.00 - j6.00	-0.00 + j4.00
3	3	-0.00 + j5.00	-0.00 + j4.00	0.00 - j9.00

- d) Solve the load flow problem for the system in Figure 1 with Newton-Raphson by writing the Jacobian matrix. Compare  $J_0$  with:

$$J_0 = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$x = \begin{bmatrix} \theta_2 \\ \theta_3 \\ V_3 \end{bmatrix} \quad f(x) = \begin{bmatrix} P_2(x) \\ P_3(x) \\ Q_3(x) \end{bmatrix} \quad y = \begin{bmatrix} P_2 \\ P_3 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -0.8 \\ -0.6 \end{bmatrix}$$

$$P_2(x) = V_2 V_1 Y_{21} \cos(\theta_2 - \theta_1 - \theta_{21}) + V_2^2 Y_{22} \cos(\theta_2 - \theta_2 - \theta_{22}) + V_2 V_3 Y_{23} \cos(\theta_2 - \theta_3 - \theta_{23})$$

$$P_3(x) = V_3 V_1 Y_{31} \cos(\theta_3 - \theta_1 - \theta_{31}) + V_3 V_2 Y_{32} \cos(\theta_3 - \theta_2 - \theta_{32}) + V_3^2 Y_{33} \cos(\theta_3 - \theta_3 - \theta_{33})$$

$$Q_3(x) = V_3 V_1 Y_{31} \sin(\theta_3 - \theta_1 - \theta_{31}) + V_3 V_2 Y_{32} \sin(\theta_3 - \theta_2 - \theta_{32}) + V_3^2 Y_{33} \sin(\theta_3 - \theta_3 - \theta_{33})$$

$$V_1 = V_2 = 1,$$

$$\theta_{11} = \theta_{22} = \theta_{33} = -\pi/2,$$

$$\theta_{ij} = \pi/2$$

$$P_2(x) = 2 \cos(\theta_2 - \pi/2) + V_3 4 \cos(\theta_2 - \theta_3 - \pi/2) = 2 \sin \theta_2 + 4 V_3 \sin(\theta_2 - \theta_3)$$

$$P_3(x) = V_3 5 \cos(\theta_3 - \pi/2) + V_3 4 \cos(\theta_3 - \theta_2 - \pi/2) = 5 V_3 \sin \theta_3 + 4 V_3 \sin(\theta_3 - \theta_2)$$

$$Q_3(x) = V_3 5 \sin(\theta_3 - \pi/2) + V_3 4 \sin(\theta_3 - \theta_2 - \pi/2) + V_3^2 9 \sin(\pi/2) \\ = -5 V_3 \cos \theta_3 - 4 V_3 \cos(\theta_3 - \theta_2) + 9 V_3^2$$

$$J = \frac{\partial f_i}{\partial x_j} = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$

$$J_{22} = \frac{\partial P_3}{\partial \theta_3} = 5V_3 \cos \theta_3 + 4V_3 \cos(\theta_3 - \theta_2)$$

$$J_{23} = \frac{\partial P_3}{\partial V_3} = 5 \sin \theta_3 + 4 \sin(\theta_3 - \theta_2)$$

$$J_{32} = \frac{\partial Q_3}{\partial \theta_3} = 5V_3 \sin \theta_3 + 4V_3 \sin(\theta_3 - \theta_2)$$

$$J_{33} = \frac{\partial Q_3}{\partial V_3} = -5 \cos \theta_3 - 4 \cos(\theta_3 - \theta_2) + 18V_3$$

$$J_{11} = \frac{\partial P_2}{\partial \theta_2} = 2 \cos \theta_2 + 4V_3 \cos(\theta_2 - \theta_3)$$

$$J_{12} = \frac{\partial P_2}{\partial \theta_3} = -4V_3 \cos(\theta_2 - \theta_3)$$

$$J_{13} = \frac{\partial P_2}{\partial V_3} = 4 \sin(\theta_2 - \theta_3)$$

$$J_{21} = \frac{\partial P_3}{\partial \theta_2} = -4V_3 \cos(\theta_3 - \theta_2)$$

$$J_{31} = \frac{\partial Q_3}{\partial \theta_2} = -4V_3 \sin(\theta_3 - \theta_2)$$

Initial iteration:  $\theta_2 = \theta_3 = 0, V_3 = 1$

$$J_0 = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

STEP1:

$$\begin{bmatrix} \Delta P_2(i) \\ \Delta P_3(i) \\ \Delta Q_3(i) \end{bmatrix} = \begin{bmatrix} P_2 - P_2(x(i)) \\ P_3 - P_3(x(i)) \\ Q_3 - Q_3(x(i)) \end{bmatrix} = \begin{bmatrix} 0.6 - P_2(x(i)) \\ -0.8 - P_3(x(i)) \\ -0.6 - Q_3(x(i)) \end{bmatrix}$$

STEP2:

$$J = \frac{\partial f_i}{\partial x_j} = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \frac{\partial P_2}{\partial V_3} \\ \frac{\partial P_3}{\partial \theta_2} & \frac{\partial P_3}{\partial \theta_3} & \frac{\partial P_3}{\partial V_3} \\ \frac{\partial Q_3}{\partial \theta_2} & \frac{\partial Q_3}{\partial \theta_3} & \frac{\partial Q_3}{\partial V_3} \end{bmatrix}$$

STEP3:

$$[J(i)] \bullet \begin{bmatrix} \Delta \theta_2(i) \\ \Delta \theta_3(i) \\ \Delta V_3(i) \end{bmatrix} = \begin{bmatrix} \Delta P_2(i) \\ \Delta P_3(i) \\ \Delta Q_3(i) \end{bmatrix}$$

Solve it by Gauss elimination and back substitution.

STEP4:

$$\begin{bmatrix} \theta_2(i+1) \\ \theta_3(i+1) \\ V_3(i+1) \end{bmatrix} = \begin{bmatrix} \theta_2(i) \\ \theta_3(i) \\ V_3(i) \end{bmatrix} + \begin{bmatrix} \Delta \theta_2(i) \\ \Delta \theta_3(i) \\ \Delta V_3(i) \end{bmatrix}$$

For  $i=0$ :

STEP1:

$$\begin{bmatrix} \Delta P_2(0) \\ \Delta P_3(0) \\ \Delta Q_3(0) \end{bmatrix} = \begin{bmatrix} 0.6 \\ -0.8 \\ -0.6 \end{bmatrix}$$

STEP2:

$$J_0 = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

STEP3:

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \bullet \begin{bmatrix} \Delta \theta_2(0) \\ \Delta \theta_3(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} \Delta P_2(0) \\ \Delta P_3(0) \\ \Delta Q_3(0) \end{bmatrix}$$

$$\begin{bmatrix} \Delta \theta_2(0) \\ \Delta \theta_3(0) \\ \Delta V_3(0) \end{bmatrix} = \begin{bmatrix} 0.06 \\ -0.06 \\ -0.07 \end{bmatrix}$$

STEP4:

$$\begin{bmatrix} \theta_2(1) \\ \theta_3(1) \\ V_3(1) \end{bmatrix} = \begin{bmatrix} 0.06 \\ -0.07 \\ 0.93 \end{bmatrix}$$

- Select Power Flow Jacobian at “Other” in the Case information menu.
- Compare it to the solution in “d”. You will find that three rows and columns agree, but that PowerWorld has a row and column for voltage magnitude at the generator bus.

**4. Load flow iterations in PowerWorld:**

- a) Select “Just One Iteration” in the Solution Environment Options. Push “Log”.
- b) Push single solution. Write down the mismatches. Compare it with your solution.

- c) Repeat until mismatch is less than 0.1 MVA. How many iterations are needed?
- d) Remove single iteration. Start continuous simulation by pushing Play in the Simulation menu and see how load calculations are done repeatedly. Try operating circuit breakers and see that log output is changing.

#### **5. Larger system in PowerWorld:**

- Run **Example 6.9** to see a slightly larger system. Note that the Ybus matrix has a number of zeros. This is called a sparse matrix. By not storing zeros and skipping multiplications by zero, computational performance can be improved. This is called sparsity techniques.
- Change transformer symbols into circles: In the Option/Tools menu, select Solution/Environment where the choice is done on the Environment tab.

#### **6. Jacobian of a larger system in PowerWorld:**

- Run **Example 6.11**. In particular read the motivation for the existence of rows in the Jacobian that corresponds to controlled voltage. When trying to reach the desired voltage the maximum or minimum reactive output limit of the generator may be hit. The generator then stays at its limit, with a known reactive output. At the same time voltage control is lost so voltage must be computed. The generator bus then behaves like a load bus and an equation for reactive power is needed. The voltage row in the Jacobian is thus there to prepare for this by reserving space in the matrix for a possible switch from PV to PQ bus.
- Change the MVAR limit so that it is activated. Verify that a voltage row in the Jacobian has changed into a reactive power row.

#### **7. Changing load flow in PowerWorld:**

- Run **Example 6.12**. Write down the maximum and minimum values of MW generation at bus Three where 100% load is reached in a transformer or a line.
- Repeat **Example 6.12** but with one line out of service: Increase and decrease the MW output of the generator at bus Three until 100% load is reached somewhere.

#### **8. Cascading outages in PowerWorld: Open the case called Cascade.**

- Run the system and try disconnecting one line at a time, and then more lines.