Engineering; Mathematics; Modeling and Applications

Engineering Concepts

What is Engineering?

Derived from the Latin ingenium, **engineering** means something like brilliant idea, flash of genius. The word was created in the 16th century and originally described a profession that we would probably call an artistic inventor. Engineering combines applied mathematical, scientific, and technical principles, to yield tangible end products which can be made, produced, and constructed. Engineering differs from science in that it must take requirements into account including costs, safety, performance and limitations on resources. All engineering work is regulated by safety standards, and issues of patents and design protection may also arise.

The art of engineering is to take a bright idea and, using money, materials, knowledgeable people, with regard for the environment, to produce something the buyer wants at an affordable price. There are a number of disciplines of engineering (chemical, civil, mechanical, electrical, etc.) which apply to different areas of design and construction.

What is an Engineer?

The term **engineer** means talent, genius, cleverness, or native ability. An engineer is a person who uses science, mathematics, experience, and judgment to create, operate, manage, control, or maintain devices, mechanisms, processes, structures, or complex systems, and who does this in a rational and economic way. Although science and mathematics are the basis of engineering knowledge, real projects require the human skills of leadership, management, and communication.

Engineers are builders and problem solvers who provide the link between theory and practice. The engineer must be a scientist and a mathematician. Additionally, the engineer must be creative and have the ability to lead a team toward a common goal. The emphasis in engineering is in making or operating things, so an engineer is free to gain and use any technique from any field that helps to carry out his task.

Engineering Disciplines

There are a number of disciplines of engineering which apply to different areas of design and construction. Most people are aware of few disciplines of engineering: civil, electrical, mechanical, and chemical engineering. However, the number of disciplines is much larger than is commonly known. The following are few of the traditional and emerging engineering disciplines:

- Aeronautical and Aerospace
- Agricultural
- Biological
- Biomedical
- Chemical
- Civil
- Electrical and Electronics
- Computer Hardware
- Computer Software
- Engineering Mathematics
- Engineering Physics
- Environmental

- Food
- Geological
- Industrial and Control
- Hydrology and Water Resources
- Management
- Manufacturing
- Materials and metallurgical
- Mechanical
- Mechatronics
- Metallurgical and Materials
- Mining
- Nanotechnology
- Naval
- Nuclear
- Ocean
- Petroleum and Natural Gas
- Sanitary
- Systems and Industrial
- Textile
- Transportation

Based on all engineering Bachelor's degrees awarded annually, Civil, Computer, Electrical, and Mechanical Engineering are considered the big four disciplines followed by Aerospace, Biomedical, Chemical, and Industrial/Manufacturing Engineering as medium four disciplines. Rests of the disciplines are considered small or specialty.

Although an engineer may be interested primarily in one discipline or even in one area of that discipline, he/she must also be knowledgeable in other areas of that discipline or even in other disciplines that interact with this discipline. This interaction is part of what makes engineering a demanding and exciting profession.

Mathematics Concepts

Mathematics Defined

The word **mathematics** comes from the Greek (máthema) which means science, knowledge, or learning. Mathematics is commonly defined as the study of quantities and relations through the use of figures, numbers, patterns, and symbols. Even though mathematics itself is not generally considered a natural science, the specific structures that are investigated by mathematicians often have their origin in the natural sciences, regularly in physics.

The fundamental principle in mathematics is the study of measurement, properties, and relationships of quantities and sets, using numbers and symbols. Mathematical studies may be accomplished with either numerical analysis, variable analysis, or a combination of the two. Mathematics is often abbreviated to **math** (in American English) or **maths** (in British English).

Divisions of Mathematics

Mathematics crosses over the interconnected branches of the mathematics and statistics field by demonstrating how applied and pure mathematics can be used in statistics and actuarial science, or how solving certain problems may require techniques drawn from calculus and algebra. In general, the study of mathematics may embrace the following subjects:

- Arithmetic
- Algebra
- Geometry
- Analysis
- Calculus
- Statistics and Probability

There are other subjects within mathematics, and many subsubjects under the major subjects.

There are two major divisions of mathematics: pure and applied. Pure Mathematics is the mathematics which underlies all applications. **Applied mathematics** is a branch of mathematics that concerns itself with the application of mathematical knowledge to other domains. Such applications include mathematics of engineering, numerical analysis, optimization, modeling, mathematical biology, bioinformatics, financial mathematics, etc. Engineering mathematics describes physical processes, and so is often indistinguishable from theoretical physics. Important subdivisions include: fluid dynamics, mechanics, and Maxwell's equation that govern electromagnetism.

Mathematics Needed By Engineers

Mathematics is present in all disciplines of science and engineering. Traditionally, it has been the needs of the physical sciences including engineering which have driven the development of many parts of mathematics, particularly analysis. Mathematical talent and problem-solving ability is needed in engineering, physics, chemistry and other sciences. In fact, with the application of mathematics in business and the social sciences, there is barely a field that does not need a good background in mathematics.

An understanding of key mathematical concepts together with a skill to apply them effectively to solve engineering problems is an essential ability that every engineering student must acquire. Traditionally, mathematics has always been taught to year 1 engineering students at most of the universities by lecturers from Departments of Mathematics and a formal, traditional teaching style has dominated. Lectures present theory and students practice by solving problems. Although this method of teaching may meet the needs of students with high competence in mathematics, formal lectures do not appear to be the most effective method for teaching mathematics to engineering students for several reasons. Many students learn to solve theoretical problems without being able to apply that knowledge and further, are exposed to pure rather than applied mathematics. As well, the teachers' perception of mathematics clearly affects the manner in which it is presented and in turn, affects students' perceptions and understanding of mathematics. The following are topics in mathematics which are extremely important for engineering:

Arithmetic
Fractions
Decimal Numbers
Percentage and Ratio
Basic Algebra
Functions
Polynomial Equations
Logarithms and Exponentials
Trigonometry
Matrices and Determinants
Systems of Linear Equations
Vectors
Complex Numbers
Differentiation
Integration
Sequences and Series
Differential Equations
Functions of More than One Variable and Partial Differentiation
The Laplace transforms
Statistics and probability
Fourier Series and the Fourier Transform

Mathematical Modeling

Many educators in engineering have long realized that they have a major stake in the quality of science and mathematics teaching at the K-12 level. Students who are deficient or lack interest in mathematics and science are unlikely to consider engineering as a career. In addition, there seem to be many reasons for students in early engineering studies dropping-out many of their courses. For them first, engineering is unfamiliar area to most entrants and accordingly expectations are not always met. Second, engineering is challenging and requires fluency in mathematics and ability to integrate mathematics with physics and other sciences to understand the subject properly. Such kind of integration whatsoever between mathematics and science is called **modeling**.

As the lack of student interest in pursuing careers that require expertise in mathematics and science has become more apparent, a number of engineering schools, like the University of Ottawa, have entered the fray, developing a summer program that educates mathematical modes and applications to K-12 students moving to engineering programs. Other universities introduced variety of initiatives to help classroom teachers do a better job of engaging students. Some of the engineering schools send students into the high school classroom to work with teachers and students, while others bring teachers to university campuses where they hone their skills and upgrade their knowledge under the guidance of engineering faculty.

Modeling has become endeavours central to all disciplines of science and engineering. Engineers use mathematical models, such as sets of equations, to analyze the behaviour of physical systems. A **physical quantity** is a measure of some quantifiable aspect of the modeled world. Physical quantities come in several types, such as the mass of a body (a scalar quantity), the displacement of a point on the body (a vector quantity), the altitude of the particle as a function of time (a scalar function quantity), and the stress at a particular point in a deformed body. Although we use the term "physical quantity" for this generalized notion of quantitative measure, the definition allows for non-physical quantities such as amounts of money or rates of inflation.

Models are also important to the design of new engineering systems where they enable us to predict the behaviour of a system before it is actually built. The modeling of any physical system requires intimate knowledge of the physical processes involved. The designer must decide what processes are to be modeled and how detailed an analysis is required. Approximations made in the modeling process can greatly alter the calculated functional behaviour. Therefore, each simplifying assumption must be justified.

Mathematical models are a way to represent reality through the development of mathematical relationships. This conceptualization of reality can be applied, for example, on modeling of stock market behaviour, population growth, weather forecasting, performance of a machine, building structure, etc. It can be applied (with varying success) to any process which can be represented by a mathematical expression. However, models are not the real things and regularly comprise some degree of simplification and approximation. Elimination of processes or elements known to be functioning in a system is often excused by assuming that these things have insignificant effects on the system as a whole. The development of mathematical models also involves the assumption that people understand the various processes well enough to be able to translate system behaviour into mathematical expressions.

Choosing a model may present some difficulty since each model is an alternate reality. Few features of effective models that can help in making an effective choice include simplicity, agreement with known facts, explanatory power and predictive capability. Models are tools for organizing reality, for ordering experiences rather than simply describing them. Sometimes, however, the data are allowed to become the model, so that there is a different model, a new and confusing alternate reality, for every set of data collected.

Mathematical models provide a concise and objective description of complex dynamic processes by defining, through mathematical equations, the relationships between quantitative measurements. Mathematical models can be developed from simple non-compartmental representations to large scale multi-compartmental models. The basic steps in the formulation of a model include conceptualization, realization and solution of the model. The modeling process is usually started with a definition of the problem and a parameter identification followed by the setting up of a clear conceptual model of the system.

This Engineering Summer School is conducted to show how mathematics naturally arises in engineering examples. We chose to focus first on high-school physics and mathematics because these two topics are related to each other. Most lessons will encourage the reader to integrate mathematics with science and engineering to generate models that motivate some aspect of learning.

Modeling and Applications

The term "modeling and applications" has been increasingly used to denote all kinds of relationships whatsoever between the real world applications and mathematics. Using mathematics to solve real world problems is often called **applying mathematics**, and a real world situation which can be tackled by means of mathematics is called an 'application' of mathematics. The term "modeling", on the other hand, is the process of representing the behaviour of a real system by a collection of mathematical equations. That means it focuses on the direction from reality to mathematics while application focuses on the opposite direction from mathematics to reality.

Modeling aims at providing students with a better understanding of mathematical concepts and teaching them to formulate and solve application-oriented-problems. Mathematical models are developed to help in the understanding of physical systems. Engineers use models to represent the elements of any system. Models are generated for manufactured elements and devices in order to facilitate understanding and establish the operating characteristics of the elements and devices.

Design is a fundamental activity that distinguishes engineering from disciplines based on pure science or mathematics. Engineering design may be defined as the systematic and creative application of scientific and mathematical principles to practical ends such as the design, manufacture, and operation of efficient and economical structures, machines, processes, and systems.

Every design begins with recognition that there is some need for improvement. However, design decisions cannot be made until the problem is defined more clearly. This requires gathering more information about the problem and perhaps conducting a feasibility study. While the problem is defined, the design criteria and constraints must be identified.

Design criteria are performance standards to be met by the design, while, design constraints are limitations placed on the final design. For example, in the design of an electric motor, the efficiency and speed are design criteria; the budget limit for the design is the constraint. Where possible, the criteria and constraints must be easily measured by quantitative values, not by subjective values that rely on opinion.

The pedagogy of applications and modeling intersects the pedagogy of pure mathematics or pure science in variety of ways and requires at the same time various practices that are not part of the traditional classroom. Approaches to teaching applications and modeling vary from the use of traditional classroom to those that include a variety of innovative teaching practices ranging from the study of physical concepts and mathematical tools to modeling and simulation with emphasis on group activity. This is an excellent tool that provides students with the opportunity to develop design skills and prepare them for careers in engineering.

The design skill is essentially a repetitive process of two phases: synthesis and analysis. Through the phase of synthesis, ideas or methods to solve the problem are suggested. However, the results of each idea or method and the cost are calculated through the phase of analysis. Figure 1 shows the steps in a typical engineering process.

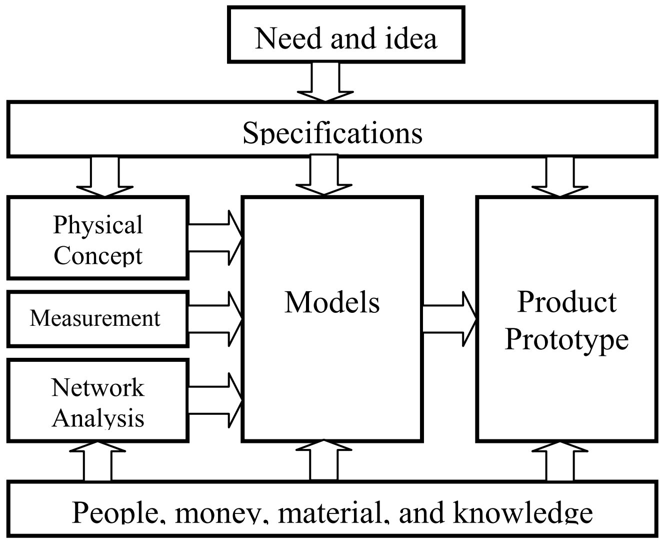

Figure 1 Steps in a typical engineering process.

Many computational tools are very appropriate for applications and modeling activities. They expand the range of options for approaches to teaching and enhance the students' experience of mathematizing situations, design and conduct simulations, and engage in applied problem solving.