# Modeling Mechanical Systems

Mechanical systems can be either translational or rotational. Although the fundamental relationships for both types are derived from Newton's law, they are different enough to warrant separate considerations.

Translational systems analysis is based on Newton's second law: the sum of all forces applied to a body equals the product of the vector acceleration of the body times its mass. The equation for Newton's second law is

$$F = ma \tag{1}$$

Where $F$ is the force in newtons (N), $m$ is the mass in grams (g), and $a$ is the acceleration in (m/s$^2$). The models used to represent translational systems have the basic building blocks of springs, dashpots, and masses. Table 1 shows the three basic building blocks and their physical representations.

**Table 1** Basic Building Blocks for Mechanical Systems

| Block | Physical Representation |
|---|---|
| Spring | Stiffness of a system. |
| Dashpot | Forces opposing motion |
| Mass | Inertial or resistance to acceleration |

A mechanical system does not have to be really made up of springs, dashpots, and masses to have the properties of stiffness, damping, and inertia. All these blocks may be considered to have a force as an input and displacement as an output.

## Spring

The stiffness of a spring is described by the relationship between the forces $F$ used to extend or compress a spring and the resulting extension or compression $x$ as shown in Figure 1.

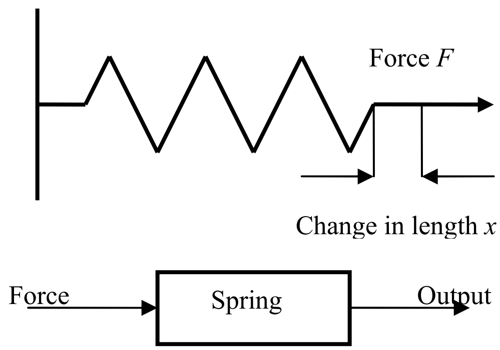


**Figure 1** Spring.

The extension or compression is proportional to the applied forces.

$$F = kx \quad (2)$$

where $k$ is a constant. The bigger the value of $k$ the greater the forces have to be to stretch or compress the spring and so the greater the stiffness. The object applying the force to stretch the spring is also acted on by a force, the force being that exerted by the stretched spring (Newton's third law). This force will be in the opposite direction and equal in size to the force used to stretch the spring.

## Dashpot

The dashpot building block represents the types of forces experienced when we endeavour to push an object through a fluid or move an object against frictional forces. The faster the object is pushed the greater becomes the opposing forces. The dashpot which is used to represent these damping forces which slow down moving objects consists of a piston moving in a closed cylinder as shown in Figure 2.

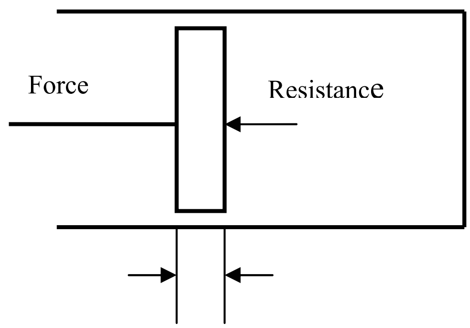


**Figure 2** Dashpot building block.

Movement of the piston requires the fluid on one side of the piston to flow through or past the piston producing a damping or resistive force. The mass experiences a damping force in the direction opposite to the instantaneous motion. For small velocities, assume that the damping force is simply proportional to the velocity of the piston with a direction opposite to the direction of motion. In the ideal case, the damping *F* is

$$F = cv \tag{3}$$

Where *c* is a constant. Since velocity is the rate of change of displacement *x* of the piston, then Equation (3) can be written as

$$F = c\frac{dx}{dt} \tag{4}$$

Therefore, the relationship between the displacement *x* of the piston and the force as the input is a relationship depending on the rate of change of the output.

## Mass

A mass building block is shown in Figure 3. The bigger the mass the greater the force required to give it a specific acceleration according to Newton's second law . Acceleration is the rate of change of velocity. Mathematically is written as

$$F = ma = m\frac{dv}{dt} = m\frac{d(dx/dt)}{dt} = m\frac{d^2x}{dt^2} \tag{5}$$

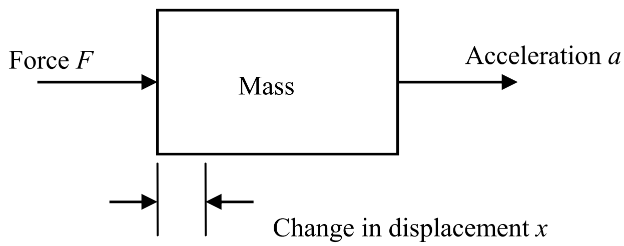


**Figure 3** Mass building block.

## Mass-Damper System

Spring forces alone are not much use though. You need to combine them with dampers to have a realistic simulation. Damping simulates energy loss, and it is used in physics simulations to make sure that springs do not oscillate forever but come to rest over time. Figure 4 shows a spring dashpot mass system. The simplest spring-damper system can be modeled as follows:

$$\mathbf{F} = -k\mathbf{x} - c\mathbf{v} \qquad (6)$$

Where $c$ is the coefficient of damping and **v** is the relative velocity between the two points connected by the spring. Larger values for $c$ increase the amount of damping so the object will come to rest more quickly, $c = 0$ will allow the spring to oscillate forever, while negative values will actually accelerate the objects over time making your simulation explode.

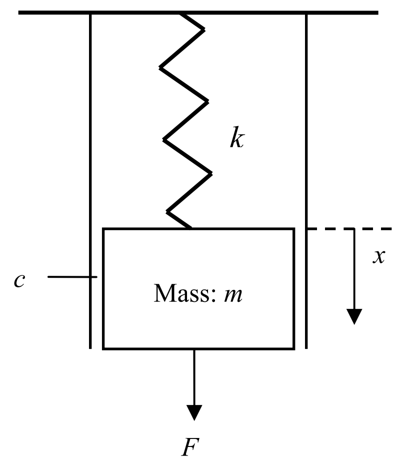


**Figure 4** Spring dashpot mass system.

Table 2 summarizes modeling equations for translational systems.

**Table 2** Translational Building Blocks

| **Building Block** | **Equation** | **Energy** |
|---|---|---|
| **Translational** | | |
| Spring | $F = kx$ | $E = \frac{1}{2}\frac{F^2}{k}$ |
| Dashpot | $F = c\frac{dx}{dt}$ | $P = cv^2$ |
| Mass | $F = m\frac{d^2x}{dt^2}$ | $E = \frac{1}{2}mv^2$ |

## Building up a Mechanical Systems

Mechanical systems are concerned with the behavior of matter under the action of forces. Such systems are categorized as rigid, deformable, or fluid in nature. A rigid-body system assumes all bodies and connections in the system to be perfectly rigid. In actual systems this is not true, and small deformation always results as various loads are applied. Subjects like mechanics of material and failure analysis consider such deformable-body systems.

Newtonian mechanics provide the basis for most mechanical systems and consist of three independent concepts: space, time, and mass. A fourth concept, force, is also considered but is not independent of the other three.

Most mechanical applications involve rigid-body systems, and the study of such systems relies on six fundamental laws as shown in Table 3.

**Table 3** Six Fundamental Laws Involving Rigid-Body Systems

| Fundamental Law | Statement |
|---|---|
| Newton's First Law | An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force. |
| Newton's Second Law | The acceleration of an object is dependent upon two variables: the net force acting upon the object and the mass of the object. |
| Newton's Third Law | For every action, there is an equal and opposite reaction. |
| Newton's Law of Gravitation | Any two objects exert a gravitational force of attraction on each other. The direction of the force is along the line joining the objects. |
| Parallelogram Law for the Addition of Forces | If we have two force vectors acting on a body and we wish to replace these two vectors with a single vector that has the same effect on the body, we can use the parallelogram law for the addition of force vectors. |
| Principle of Transmissibility | A force can be applied anywhere along its line of action, with no change in any of the effects on the body. |

Many mechanical systems can be modeled by the simple mass-spring-damper system shown in Figure 5 (a). This system is described by Newton's second law of motion. It may represent, for example, an automobile shock absorber.

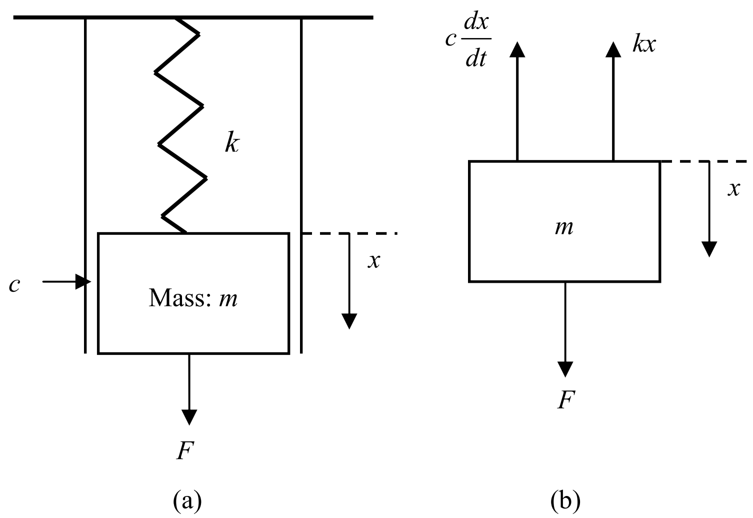


**Figure 5** (a) Spring-mass-damper system. (b) Free-body diagram.

In this example, we model the wall friction as a damper. This means the friction force is linearly proportional to the velocity of the mass. In reality, the friction force may behave in a more complicated fashion. Summing the forces acting on the mass and utilizing Newton's second law yields

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F \qquad (7)$$

Equation (7) is a second-order differential equation that describes the relationship between the input of force $F$ and the output displacement $x$. There are many systems which can be modeled using the mass-spring-damper building blocks.