ELG3175 Introduction to Communication Systems

Lecture 16

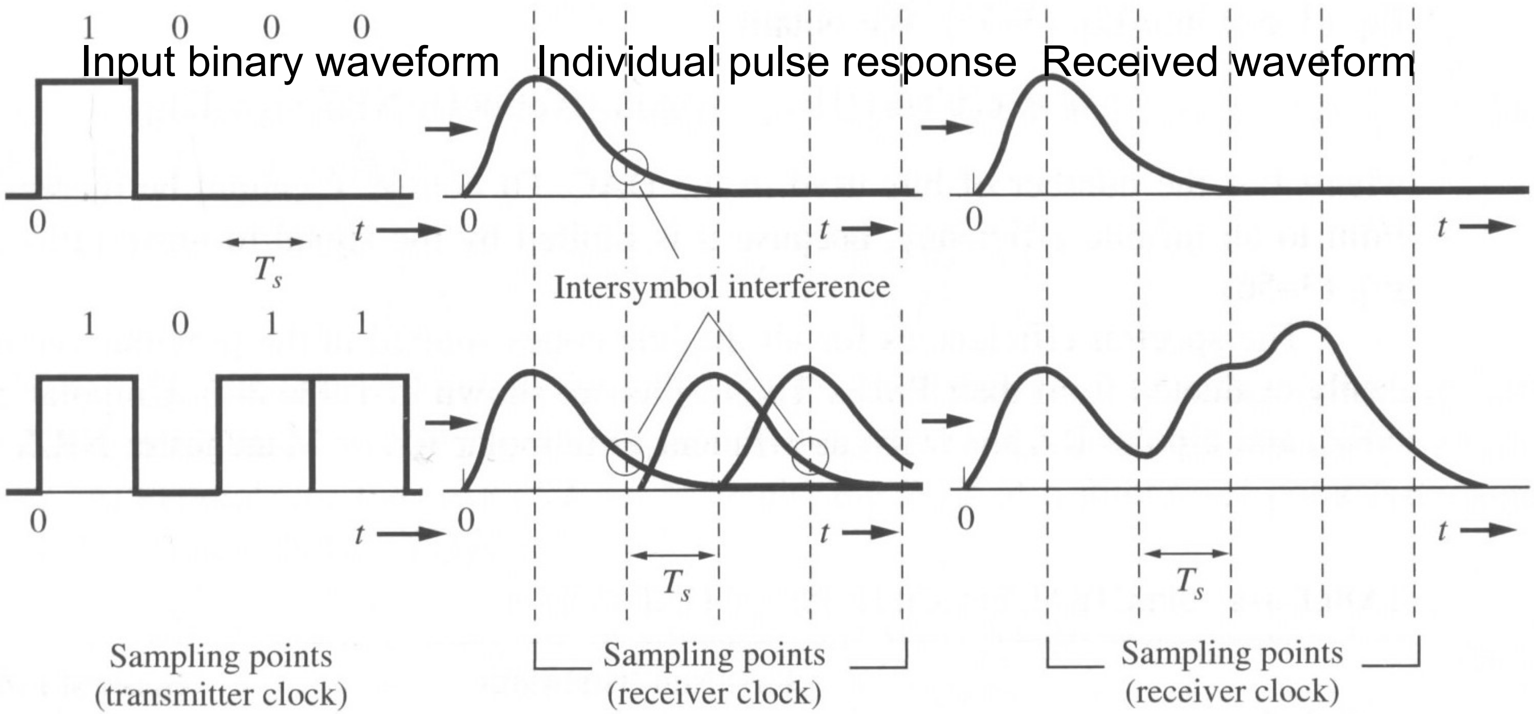
Bandlimiting and Nyquist Criterion

Bandlimiting and ISI

- **Real systems are usually bandlimited.**
- When a signal is bandlimited in the frequency domain, it is usually smeared in the time domain. This smearing results in intersymbol interference (ISI).
- The only way to avoid ISI is to satisfy the 1st Nyquist criterion.
- For an impulse response this means at sampling instants having only one nonzero sample.

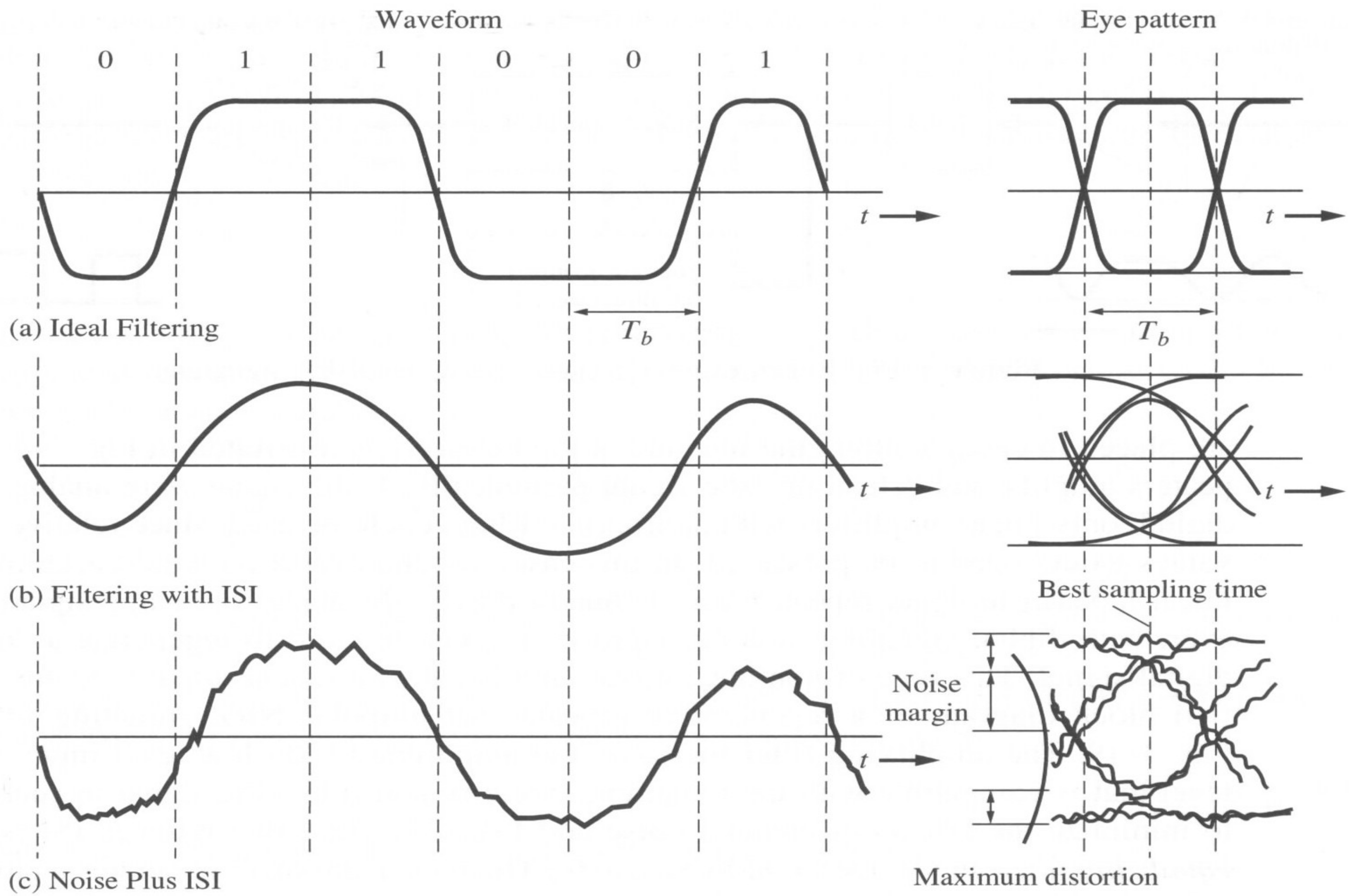
Band-limited Channels and Intersymbol Interference

- Rectangular pulses are suitable for infinite-bandwidth channels (practically – wideband).
- Practical channels are band-limited -> pulses spread in time and are smeared into adjacent slots. This is intersymbol interference (ISI).



Eye Diagram

- Convenient way to observe the effect of ISI and channel noise on an oscilloscope.



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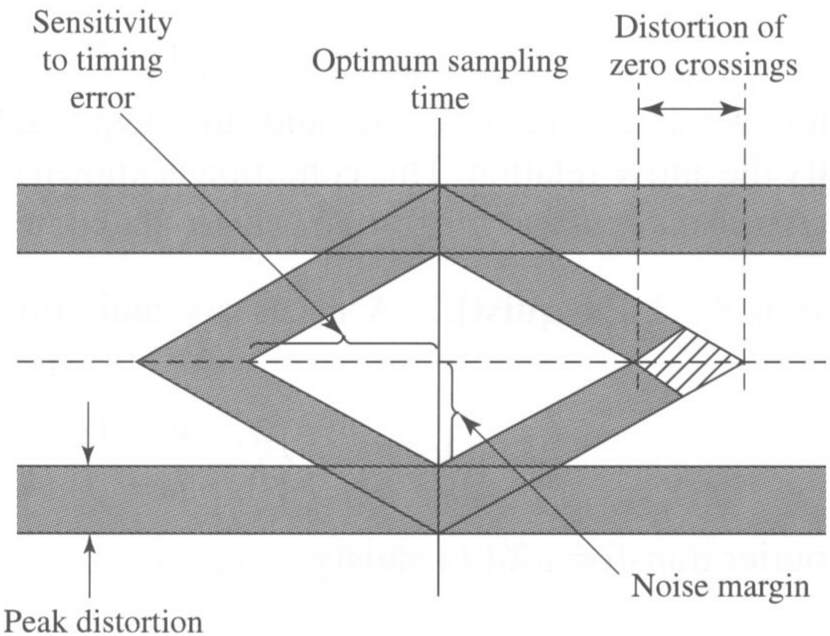
Eye Diagram

- Oscilloscope presentations of a signal with multiple sweeps (triggered by a clock signal!), each is slightly larger than symbol interval.
- Quality of a received signal may be estimated.
- Normal operating conditions (no ISI, no noise) -> eye is open.
- Large ISI or noise -> eye is closed.

- Timing error allowed – width of the eye, called eye opening (preferred sampling time – at the largest vertical eye opening).
- Sensitivity to timing error -> slope of the open eye evaluated at the zero crossing point.
- Noise margin -> the height of the eye opening.



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Pulse shapes and bandwidth

- For PAM:

$$s_{PAM}(t) = \sum_{i=0}^{L} a_i p(t - iT_s) = p(t) * \sum_{i=0}^{L} a_i \delta(t - iT_s)$$

Let $\sum_{i=0}^{L} a_i \delta(t - iT_s) = y(t)$

Then $S_{PAM}(f) = P(f)Y(f)$

$$B_{PAM} = B_p.$$

We can show that the same is true for PPM

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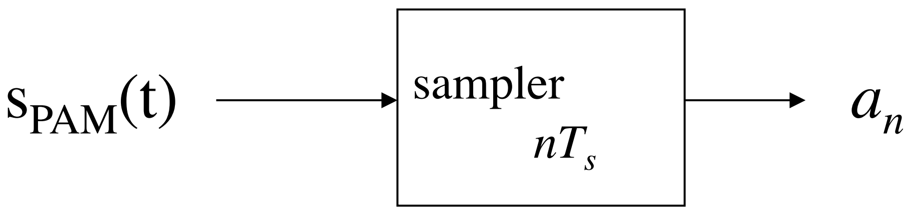
Detection of data



$$s_{PAM}(nT_s) = \sum_{i=0}^{L} a_i p(nT_s - iT_s) = \sum_{i=0}^{L} a_i p\left[(n-i)T_s\right] = a_n + \sum_{\substack{i=0 \\ i \neq n}}^{L} a_i p\left[(n-i)T_s\right]$$

The second term is Intersymbol interference (ISI)

Nyquist criterion for zero ISI

$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = nT_s \ (n \neq 0) \end{cases}$$

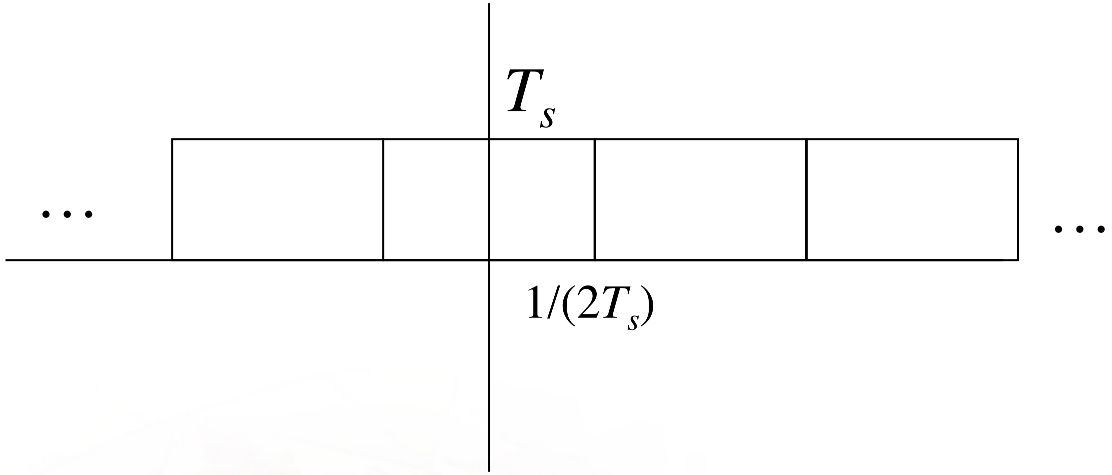
$$p_s(t) = \sum_{n=-\infty}^{\infty} p(nT_s)\delta(t - nT_s) = \delta(t)$$

$$P_S(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_s}\right) = 1$$

Therefore

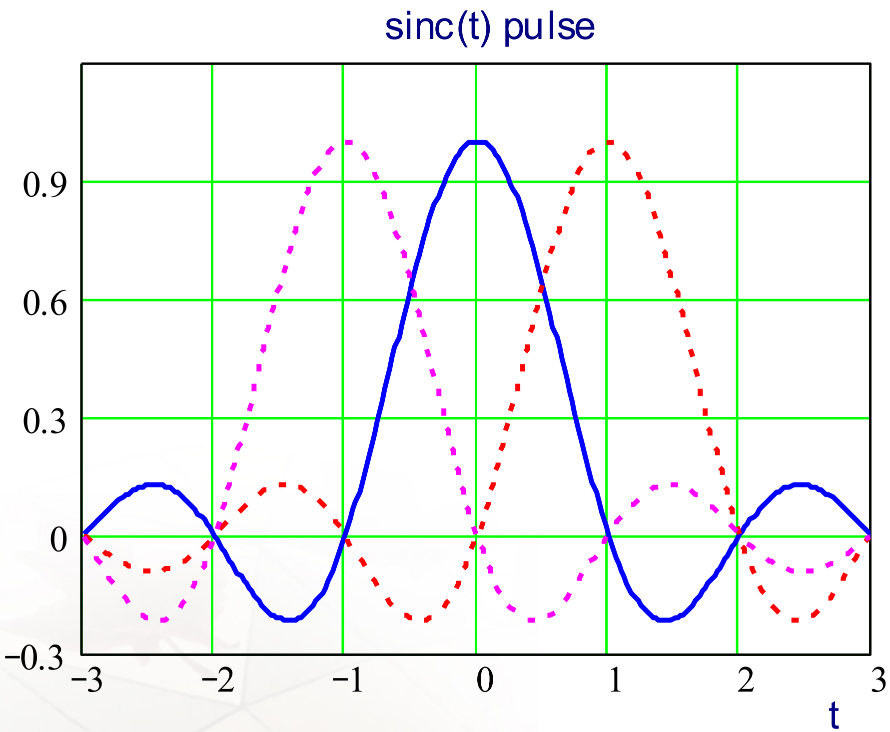
$$\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_s}\right) = T_s$$

Minimum bandwidth of PAM signal



$$P_{min}(f) = T_s \Pi(f T_s)$$
$$P_{min}(t) = \text{sinc}(t/T_s)$$

Zero ISI: sinc Pulse



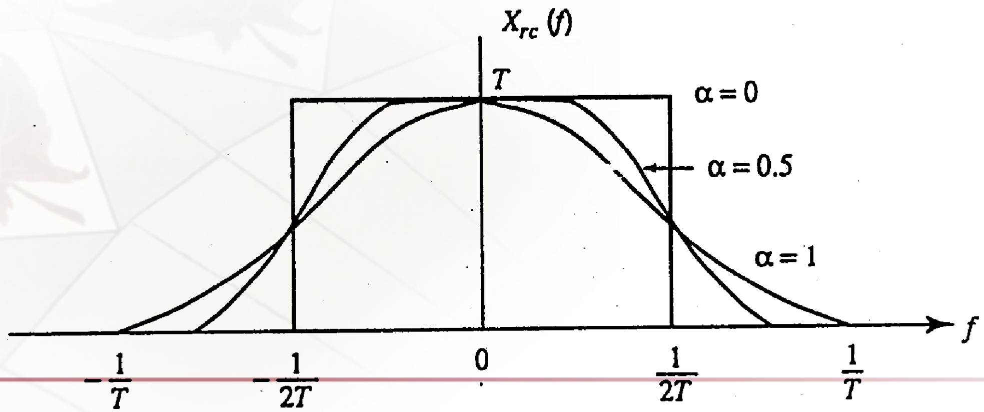
- Example: $s(t) = \mathrm{sinc}(f_0 t) \Rightarrow s(nT) = \mathrm{sinc}(n) = 0, \; n \neq 0$
- Hence, *sinc* pulse allows to eliminate ISI at sampling instants. However, it has some (2) serious drawbacks.

Pulses that satisfy Nyquist's Zero ISI Criterion

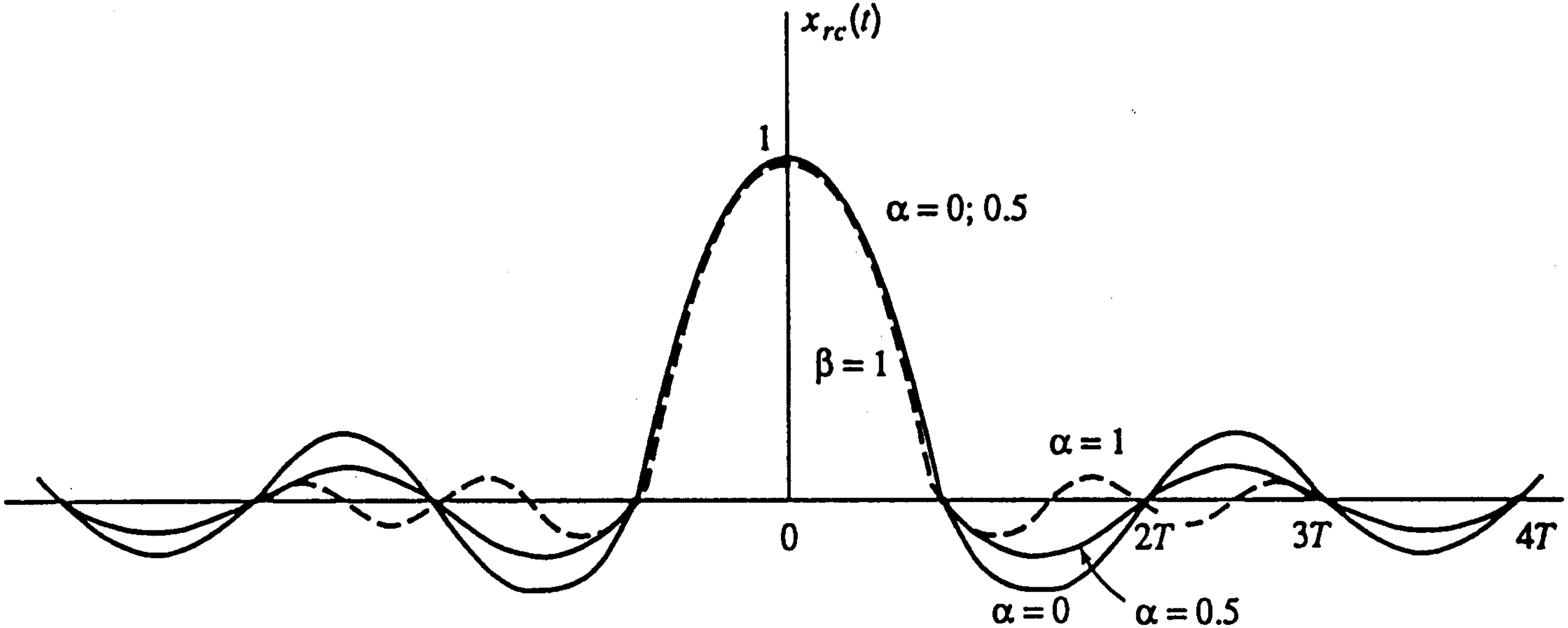
- A minimum bandwidth system satisfying the Nyquist criterion has a rectangular shape from -1/2T to 1/2T.
- Any filter that has an excess bandwidth with odd-symmetry around Nyquist frequency (1/2T) also satisfies the requirement.
- A family of such filters is known as raised cosine filters. Raised cosine pulse produces signal with bandwidth $(1/2T_s)(1+\alpha)$, where α is the roll-off factor.

Raised cosine filter transfer function

$$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le \frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1 + \cos\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right], & \frac{1-\alpha}{2T} \le |f| \le \frac{1+\alpha}{2T} \\ 0 . & |f| \ge \frac{1+\alpha}{2T} \end{cases}$$



Raised cosine filter impulse response



Equal splitting of raised-cosine characteristics